Article

Dark Energy Cosmological Model in *f*(*R*,*T*) Gravity

A. Y. Shaikh[1*] & K. S. Wankhade[2]

[1]Department of Mathematics, Indira Gandhi Mahavidyalaya, Ralegaon-445402, India
[2] Department of Mathematics, Y.C.Science College, Mangrulpir-444403, India

Abstract

The dark energy model with EoS parameter is derived for hypersurface-homogenous space-time filled with perfect fluid source in the frame work of $f(R, T)$ gravity (Harko et al., arXiv: 1104.2669v2 [gr-qc], 2011). To obtain a determinate solution, special law of variation for Hubble's parameter proposed by Berman [(Nuvo Cimento B, 74,183(1983)] is used. We have also assumed that the scalar expansion is proportional to shear and the EoS parameter is proportional to skewness parameter. In fact, the possibility of reconstruction of the hypersurface-homogenous cosmology with an appropriate choice of a function $f(T)$ has been proved in $f(R, T)$ gravity. It is observed that the EoS parameter, skewness parameters in the model turn out to be functions of cosmic time. Some physical and kinematical properties of the model are also discussed. We have also discussed the well known astrophysical phenomena, namely, look-back time, proper distance, luminosity distance, angular diameter distance with redshift.

Keywords: Dark energy, constant deceleration parameter, $f(R, T)$ gravity, hypersurface-homogenous space-time.

1. Introduction

Recent discovery of dark energy as a theoretical clarification of the accelerated growth of the Universe is a great surprise in cosmology. The proof of the existence of dark energy comes from the observations of stars (Perlmutter et al. 1997, 1998, 1999; Riess et al. 1998) and alternative observations such as cosmic microwave background (CMB) anisotropies measured with WMAP satellite (Spergel et al. 2003; Bennett et al. 2003) and enormous scale structure (Verde et al. 2002; Hawkins et al. 2003; Abazajian et al. 2004). It is estimated that dark energy occupies 73% of the energy of the universe, whereas dark matter occupies 23%, and usual baryonic matter occupy 4%. To account for these observations, various modified theories of gravity have been developed. Recently, Harko et al. (2011) modified Einstein's theory of gravitation such that the

[*] Correspondence Author: A. Y. Shaikh, Department of Mathematics, Indira Gandhi Mahavidyalaya,Ralegaon-445402, India.
E-mail: shaikh_2324ay@yahoo.com

attractive force Lagrangian is given by an discretionary operator of the Ricci scalar R and of the trace T of the strain energy tensor T_{ij}. Adhav (2012) has obtained Bianchi type-I cosmological model in $f(R,T)$ gravity. Katore and Shaikh (2012) investigated Kantowaski-Sachs cosmological models with eolotropic dark energy in $f(R,T)$ gravity. Reddy and Shanthikumar (2013) have obtained Bianchi type-II cosmological model during this modified theory of gravity. Rao and Neelima (2013) studied spatially unvaried and eolotropic Bianchitype-VI_0 reference system stuffed with perfect fluid normally scientific theory and additionally within the framework of $f(R,T)$ gravity.

A five dimensional Kaluza-Klein cosmological model is taken into account within the frame work of $f(R,T)$ gravity are investigated by Reddy et. al.(2013). A spatially unvaried and eolotropic Bianchi type-III reference system is taken into account by Kiran and Reddy (2013) within the presence of bulk viscous fluid containing one dimensional cosmic string within the frame work of $f(R,T)$ gravity. Bianchi type-V universe stuffed with dark energy (DE) from a wet dark fluid (WDF) within the framework of $f(R,T)$ gravity are studied by Samanta (2013). Kantowski-Sachs universe stuffed with perfect fluid within the framework of theory of $f(R,T)$ gravity has been derived by Samanta (2013). Reddy et al. (2013) investigated unvaried and eolotropic Bianchi type-III dark energy cosmological model in $f(R,T)$ gravity with variable EoS parameter within the presence of perfect fluid supply. Reddy and Santhi Kumar (2013) have investigated LRS Bianchi type-II cosmological models in $f(R,T)$ gravity.

A brand new category of Bianchi cosmological models in $f(R,T)$ gravity has been obtained by Chaubey and Shukla (2013). Reddy et. al. (2013) conferred a spatially unvaried and eolotropic LRS Bianchi type-II model in $f(R,T)$ gravity, once the supply of energy momentum tensor may be a viscous fluid containing one dimensional cosmic string. The Einstein-Rosen reference system stuffed with perfect fluid within the framework of $f(R,T)$ gravity has been studied by Rao and Neelima (2013). Sharma and Singh (2014) studied Bianchi type-II string cosmological model in presence of field of force within the context of $f(R,T)$ theory of gravity. Rani et. al.(2014) have explored Bianchi type-III string cosmological models with field of force in $f(R,T)$ theory of gravity. Sahoo et. al. (2014) studied axially symmetric cosmological model in $f(R,T)$ gravity. Mishra and Sahoo (2014) investigated Bianchi Type VI_h perfect fluid cosmological model in $f(R,T)$ gravity. Kaluza-Klein dark energy model in the form of wet dark fluid in $f(R,T)$ gravity have been discussed by Sahoo and Mishra (2014).

Study of Bianchi I eolotropic model in $f(R,T)$ gravity has been done by Sharif and Zubair (2014). Shamir (2014) have investigated the precise solutions of Bianchi kind I cosmological model in $f(R,T)$ gravity. Ahmed and Pradhan (2014) created a brand new category of cosmological models in $f(R,T)$ gravity. M. Sharif and Z. Yousaf (2014) study the factors poignant the soundness of domestically isotropous spherical self-gravitating systems in $f(R,T)$ gravity. Yadav et. al. (2014) have thought-about $f(R,T)$ gravity model with associate in nursing discretionary coupling between matter and pure mathematics in LRS Bianchi-I reference system.

The spatially unvaried and completely eolotropic Bianchi Type-II reference system dark energy model with EoS parameter is taken into account by Singh and Sharma (2014) within the presence of an ideal fluid supply within the framework of $f(R,T)$ gravity. The dark energy model with equation of state (EoS) parameter comes by Chirde and Shekh (2015) for a non-static plane symmetric space-time system stuffed with an ideal fluid supply within the framework of $f(R,T)$ gravity. Spatially unvaried and eolotropic Bianchi type-V reference system is taken into account within the presence of cosmic strings in $f(R,T)$ gravity by Rao and Papa Rao (2015).Five dimensional Kaluza-Klein reference system is taken into account within the presence of eolotropic dark energy in $f(R,T)$ gravity by Rao and Papa Rao (2015).Sahoo and Sivakumar (2015) obtained LRS Bianchi type-I cosmological model in $f(R,T)$ theory of gravity with Λ(T).

Inspired by the above investigations and discussion, we discuss in this paper hypersurface-homogenous dark energy cosmological model, with variable EoS parameter, in $f(R,T)$ gravity by choosing an appropriate form of $f(T)$ proposed by Harko et al (2011).

2. Metric and Field equations

Stewart and Ellis (1968) discussed general solutions of Einstein's field equations for a perfect fluid satisfying a barotropic equation of state. Hajj-Boutros (1985) proposed a method to build exact solutions of field equations in case of the metric (1) in presence of a perfect fluid and obtained exact solutions of the field equations which add to the rare solutions not satisfying the barotropic equation of state. We consider the Hypersurface-homogeneous space time of the form:

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)\left[dy^2 + \sum^2 (y,K)dz^2 \right], \tag{1}$$

where $A(t)$ and $B(t)$ are the cosmic scale functions, $\sum(y,K) = \sin y, y, \sinh y$ respectively when $K = 1, 0, -1$.

Hypersurface–homogeneous cosmological models containing a bulk viscous fluid with time varying G and Λ have been presented by Shri Ram and M.K. Verma (2010).The exact solutions of the field equations for Hypersurface-homogeneous space time under the assumption on the anisotropy of the fluid (dark energy) are obtained for exponential and power-law volumetric expansions in a scalar-tensor theory of gravitation by Katore and Shaikh (2015). A class of solutions of Einstein's field equations describing two-fluid models of the universe in Hypersurface-Homogenous space time have been investigated by Katore and Shaikh (2015) .

The Energy momentum tensor for anisotropic dark energy is given by

$$T_j^i = diag[\rho, -p_x, -p_y, -p_z] = diag[1, -w_x, -w_y, -w_z], \tag{2}$$

where ρ is the energy density of the fluid and p_x, p_y, p_z are the pressure along x, y, z axis respectively.

The Energy momentum tensor may be parameterized as

$$T_j^i = diag[1, -w, -(w+\delta), -(w+\delta)]\rho. \tag{3}$$

For the sake of simplicity we choose $w_x = w$ and the skewness parameter δ are the deviations from *w* on *y* and *z* axis respectively. Now varying the action

$$S = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\,d^4x + \int L_m \sqrt{-g}\,d^4x, \tag{4}$$

of the gravitational field with respect to the metric tensor components g_{ij} we obtain the field equation of $f(R,T)$ gravity model as (Harko et. al.(2011))

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)R_{ij} + (g_{ij\rightarrow} - \nabla_i\nabla_j)f_R(R,T) = 8\pi T_{ij} - f_R(R,T)T_{ij} - f_T(R,T)\theta_{ij}, \tag{5}$$

where

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\partial(\sqrt{-g})}{\partial g^{ij}}L_m, \theta_{ij} = -2T_{ij} - pg_{ij}, \tag{6}$$

$f(R,T)$ is an arbitrary function if Ricci Scalar R and of the trace T of the stress energy tensor of matter T_{ij} and L_m is the matter Langrangian density and in the present study we have assumed that the stress energy tensor of matter as

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}. \tag{7}$$

Now assuming that the function $f(R,T)$ given by

$$f(R,T) = R + 2f(T), \tag{8}$$

where $f(T)$ is an arbitrary function of trace of the stress energy tensor of matter and using the equation (6) and (7), the field equation (5) takes the form

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} + 2f'(T)T_{ij} + [2pf'(T) + f(T)]g_{ij}, \tag{9}$$

where the overhead prime indicates differentiation with respect to argument.

We also choose

$$f(T) = \mu T, \tag{10}$$

where μ is constant.

Now assuming commoving coordinate system, the field equations (9) for the metric (1) with the help of equations (3) and (10) can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} = \rho[(8\pi + 2\mu)w - (1 - 3w - 2\delta)] - 2\mu p, \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} = \rho[(8\pi + 2\mu)(w + \delta) - (1 - 3w - 2\delta)] - 2\mu p, \tag{12}$$

$$2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} = -\rho[(8\pi + 2\mu) + (1 - 3w - 2\delta)] - 2\mu p, \tag{13}$$

where the overhead dot denotes differentiation with respect to t.

3. Solution of the field equations

The directional Hubble parameters in the directions x, y, z for the Hypersurface-Homogenous metric defined in (1) may be defined as follows:

$$H_x = \frac{\dot{A}}{A} \text{ and } H_y = H_z = \frac{\dot{B}}{B}. \tag{14}$$

The mean Hubble parameter, H, is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \quad , \tag{15}$$

where a is the mean scale factor and $V = a^3 = AB^2$ is the spatial volume of the universe.
The anisotropy parameter of the expansion Δ is defined as

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{16}$$

in the x, y, z directions, respectively. $\Delta = 0$ corresponds to isotropic expansion.

Let us introduce the dynamical scalars, such as expansion parameter (θ) and the shear (σ^2) as usual

$$\theta = 3H \, . \tag{17}$$

$$\sigma^2 = \frac{3}{2}\Delta H^2 \, . \tag{18}$$

The field equations (11)-(13) are three independent equations in six unknowns. Hence to obtain determinate solution of the system we take the help of special law of variation proposed by Bermann (1983) for Hubble's parameter that yields constant deceleration parameter models of the universe. Hence to find deterministic solution three more conditions are necessary, we consider the following conditions:

(i) We apply the variation of Hubble Parameters proposed by Berman (1983) that yields constant deceleration parameter as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = \text{constant}. \tag{19}$$

Here the constant is taken as negative since $f(R,T)$ gravity is about accelerated expansion of the universe.

(ii)We assume that the Expansion Scalar θ is proportional to the Shear Scalar σ which gives us

$$A = B^n , \tag{20}$$

where n is a constant .

The motive behind this assumption is explained by Collins et al. (1980),Katore and Shaikh (2014a,2014b,2014c,2015).

(iii) The EoS parameter w is proportional to skewness parameter δ such that

$$w+\delta=0. \tag{21}$$

The solution of equation (19) is given by

$$a=[(\alpha_1 t+\alpha_2)]^{\frac{1}{m}}, \tag{22}$$

where $\alpha_1 \neq 0$ and α_2 be the constants of integration.

Now using equations (19), (20), and (22) the expansion for the metric coefficient in the field equations are

$$A=[(\alpha_1 t+\alpha_2)]^{\frac{3n}{m(n+2)}}, \tag{23}$$

$$B=[(\alpha_1 t+\alpha_2)]^{\frac{3}{m(n+2)}}. \tag{24}$$

With the suitable choice of coordinates and constants, the metric (1) with the help of (23) and (24) can be

$$ds^2=dt^2-(\alpha_1 t+\alpha_2)^{\frac{6n}{m(n+2)}}dx^2-(\alpha_1 t+\alpha_2)^{\frac{6}{m(n+2)}}\left[dy^2+\sum^2(y,K)dz^2\right]. \tag{25}$$

The model (25) has no initial singularity.

4. Some Physical properties of the model

Equation (25) represents Hypersurface-Homogenous Dark Energy Model in *f(R,T)* gravity with the following physical and kinematical parameters of the model which are important for discussing the physical of the cosmological model.

The Spatial volume in the model is

$$V=(\alpha_1 t+\alpha_2)^{\frac{3}{m}}. \tag{26}$$

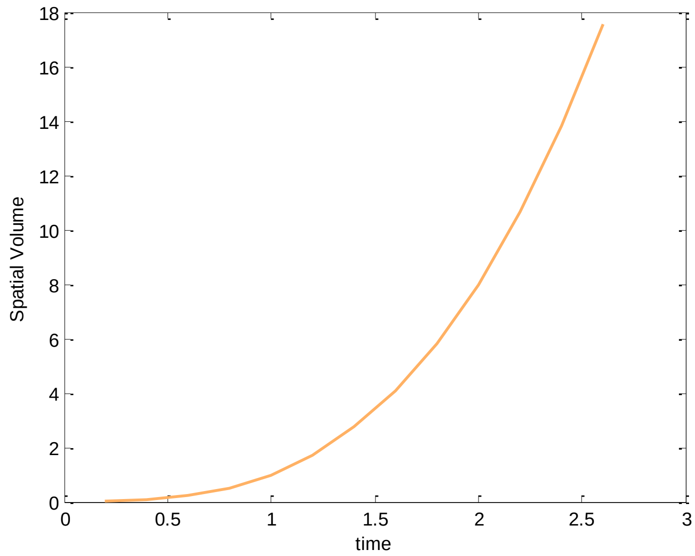

Figure 1. Spatial Volume vs time.

The spatial volume increases with time giving the accelerated expansion of the universe as shown in figure 1 which resembles with Katore and Shaikh(2012). At $t = -\frac{-\alpha_2}{\alpha_1}$, the volume element of the model vanishes.

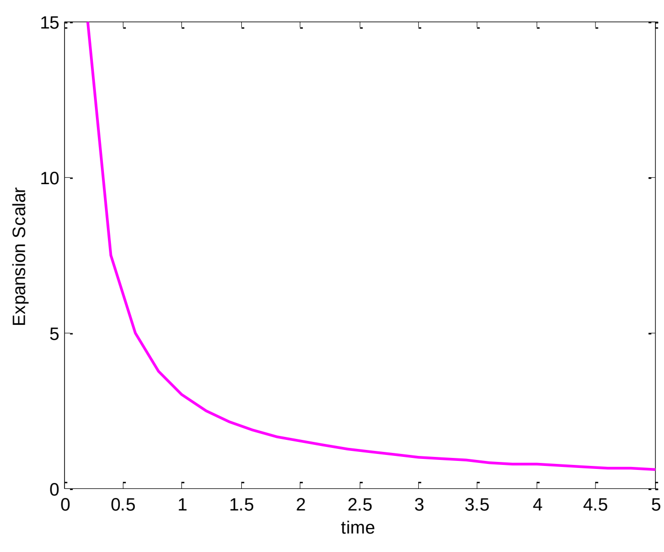

Figure 2. Expansion Scalar vs time.

The generalized Hubble Parameter is

$$H = \frac{\alpha_1}{m(\alpha_1 t + \alpha_2)} . \tag{27}$$

At an initial epoch, i.e., as $t \to 0$, the Hubble parameter tends to infinite values whereas $H \to 0$ as $t \to \infty$.

The Scalar expansion in the model is

$$\theta = \frac{3\alpha_1}{m(\alpha_1 t + \alpha_2)} . \tag{28}$$

As in figure 2, the Shear Scalar has infinite large values as $t \to 0$, whereas with the growth of cosmic time it decreases to null values as $t \to \infty$.
Mean anisotropy parameter is

$$\Delta = \frac{3(n^2+2)}{(n+2)^2} . \tag{29}$$

The mean anisotropic parameter is uniform throughout the evolution of the universe, since it does not depend on the cosmic time *t* which resembles with the investigations of Katore et. al. (2011)

\The Shear Scalar in the model is

$$\sigma^2 = \frac{9(n^2+2)}{2(n+2)^2} \frac{\alpha_1^2}{m^2(\alpha_1 t + \alpha_2)^2} . \tag{30}$$

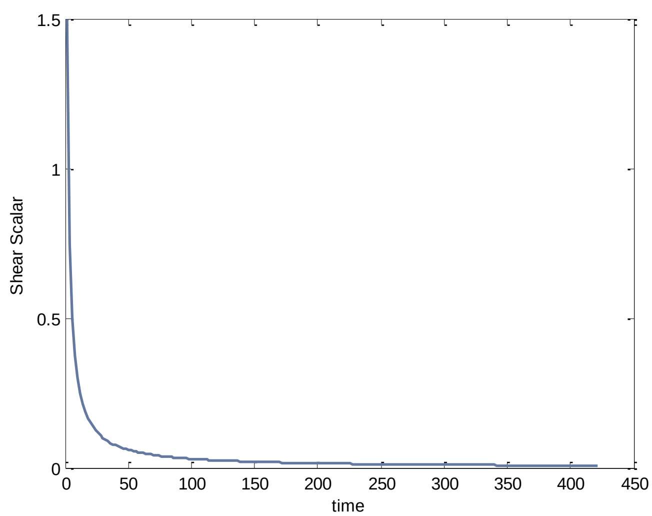

Figure 3. Shear Scalar vs time.

In the beginning of the universe, i.e., as $t \to 0$,the shear scalar assumed infinitely large value whereas with the growth of cosmic time it decreases to null values as $t \to \infty$ as shown in figure 3.

The energy density in the model is

$$\rho = \frac{1}{(8\pi + 2\mu)} \left\{ \frac{3\alpha_1^2 (n+1)(3 - mn - 2m) + 9\alpha_1^2 (n^2 - 2n - 1)}{m^2 (n+2)^2 (\alpha_1 t + \alpha_2)^2} - \frac{K}{[(\alpha_1 t + \alpha_2)]^{\frac{6}{m(n+2)}}} \right\} = -p \qquad (31)$$

Since in the case of accelerated expansion we have $\rho + p = 0$ (Reddy et. al. (2013))

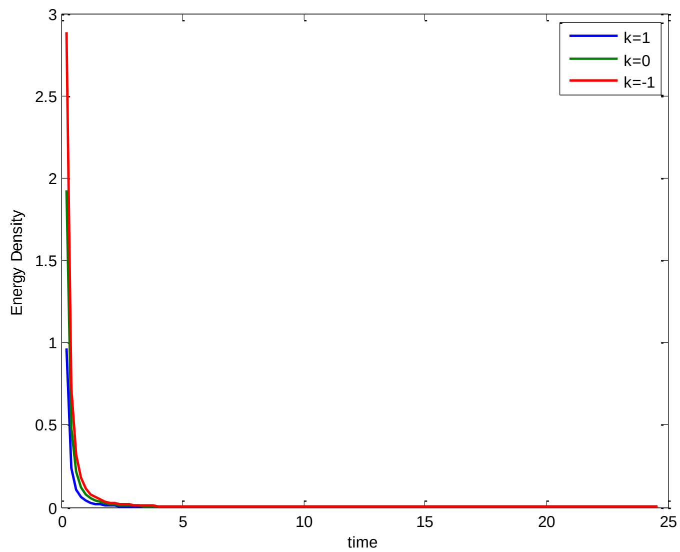

Figure 4. Energy Density vs time .

We see in Fig. 4, ρ decreases with respect to time which resembles with Katore et. al. (2010)
The Equation of state and Skewness parameter in the model are

$$w = \frac{1}{\rho(8\pi + 2\mu)} \left\{ \frac{3\alpha_1^2 (3 - mn - 2m) - 3n\alpha_1^2 (3n - mn - 2m) - 9n\alpha_1^2}{m^2 (n+2)^2 (\alpha_1 t + \alpha_2)^2} + \frac{K}{[(\alpha_1 t + \alpha_2)]^{\frac{6}{m(n+2)}}} \right\} = -\delta \qquad (32)$$

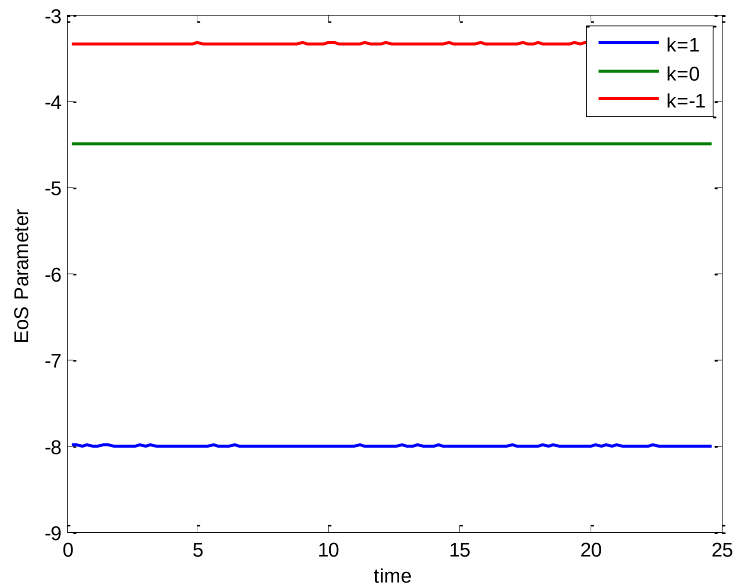

Figure 5. EoS parameter vs time.

It is observed that EoS parameter, skewness parameters in the model are all functions of t. Figure 5 depicts the variation of EoS parameter (ω) versus cosmic time (t) in evolution of the universe, as a representative case with appropriate choice of constants of integration and other physical parameters using reasonably well known situations. We observed that for $K = -1,0,1$, there is phantom region ($\omega < -1$) always (i.e. at initial as well as late time) , which resembles with the investigations of Pradhan et. al.(2012).

5. Some observational parameters

In this section, we investigate the consistency of our models with the observational parameters. We measure the physical parameters such as redshift, look-back time, luminosity distance, distance modulus.

5. (a) Look back time redshift:

The expansion scale factor $a(t)$ and the redshift z are related through the equation

$$(1+z) = \frac{a_0}{a}, \tag{33}$$

where a_0 represent the present value of scale factor.

Now we solve the equations (22) and (33) we get

$$1+z=\frac{a_0}{a}=\left(\frac{\alpha_1 t_0+\alpha_2}{\alpha_1 t+\alpha_2}\right)^{1/m}, \text{for } m\neq 0. \tag{34}$$

Above equation gives

$$(\alpha_1 t+\alpha_2)=(\alpha_1 t_0+\alpha_2)(1+z)^{-m}. \tag{35}$$

This equation can also be expressed as

$$H_0(t_0-t)=\frac{1}{m}\left[1-(1+z)^{-m}\right], \tag{36}$$

where H_0 is the Hubble's constant at present.

For the small value of redshift *z*, above equation reduces to

$$H_0(t_0-t)=\frac{1}{m}\left[mz-\frac{m(m-1)}{2}z^2+\ldots\ldots\ldots\right]. \tag{37}$$

Using equation (19) we get

$$H_0(t_0-t)=\left[z-\frac{q}{2}z^2+\ldots\ldots\ldots\right], \tag{38}$$

for $m=\frac{3}{2}$, we get the well-known Einstein de-Sitter result

$$H_0(t_0-t)=\frac{2}{3}\left[1-(1+z)^{-3/2}\right], \tag{39}$$

5. (b) Luminosity distance:

The luminosity distance is described by the simple expression

$$d_L=r_1(1+z)a_0, \tag{40}$$

for the determination of r_1, we assume that a photon emitted by source with coordinate $r=r_0$ to $t=t_0$ and received at a time *t* by an observer located at $r=0$, then we determine r_1 from following relation

$$r_1=\int_t^{t_0}\frac{dt}{a}=\int_t^{t_0}\frac{dt}{(\alpha_1 t+\alpha_2)^{1/m}}. \tag{41}$$

Hence, using above equation (41) we get the expression for luminosity distance as

$$d_L=\frac{(1+z)H_0^{-1}[1-(1+z)^{1-n}]}{(n-1)}. \tag{42}$$

5. (c) Distance modulus:

The distance modulus universe is given by the simple expression

$$\mu(z)=5\log d_L(z)+25, \tag{43}$$

where $d_L(z)$ is the luminosity distance.

From equation (42)

$$\mu(z) = 5\log\left\{\frac{(1+z)H_0^{-1}[1-(1+z)^{1-n}}{(n-1)}\right\} + 25\,. \tag{44}$$

The distance modulus of derived model is in good agreement with SN Ia data.

5. (d) Cosmic Jerk parameter:

The jerk parameter j in cosmology could be a convenient methodology to explain models near ΛCDM. The jerk parameter $j(t)$ is outlined as a dimensionless third spinoff of the scale factor a with relation to time (Chiba and Nakamura 1998; Visser 2004, 2005;Sahni 2002; Blandford et al. 2004).

$$j = \frac{\dddot{a}}{H^3 a} = \frac{\left(a^2H^2\right)''}{2H^2}, \tag{45}$$

where dot and prime denotes differentiation with respect to time and scale factor respectively. The above equation can be written as

$$j = q + 2q^2 - \frac{\dot{q}}{H}\,. \tag{46}$$

The deceleration to acceleration transition occurs for models with a positive value of j_0 and negative value of q_0.

Hence using equation (15) and (19) we get

$$j = (1-m)(1-2m)\,. \tag{47}$$

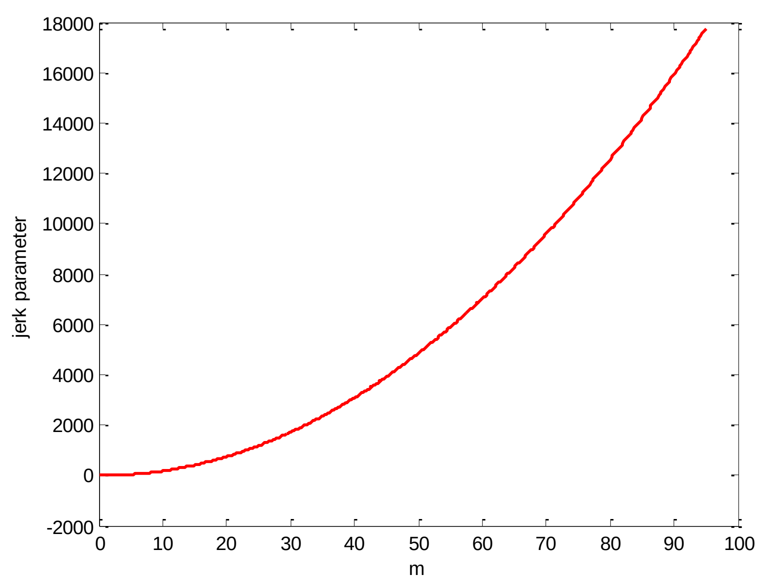

Figure 6. Jerk parameter vs m.

This value overlaps with Flat ΛCDM models $j = 1$ for $m = 3/2$ which resembles with the investigations of Chaubey and Shukla (2012). Figure 6 shows that the jerk parameter is positive throughout the entire history of the universe and for large cosmic time, the jerk parameter is greater than 1.This physical behavior resembles with the behavior of Sarkar (2014).

5. (e) State finder diagnostics:

A new geometrical diagnostic, dubbed the statefinder combine {r,s} is planned by Sahni et al. (2003), wherever r is merely determined by scalar factor a and its derivatives with reference to the time t , even as the Hubble parameter H and also the speed parameter q, and s could be a straightforward combination of r and q. The two parameters have a great geometrical significance since they are derived from the cosmic scale factor alone. Its important property is that {r, s} = {1, 0} is a fixed point for the flat ΛCDM FRW cosmological model. The statefinder parameters can effectively differentiate between different form of dark energy and provide simple diagnosis regarding whether a particular model fits into the basic observational data. The statefinder combine has been wont to explore a series of dark energy and cosmological models (Li et al. 2009;Liu and Liu 2008; Huang and Lu, H.Q 2008; Jamil and Debnath2011; Katore and Shaikh(2012)).

The pair of state finder diagnostic has a following form

$$r = 1 + 3\frac{\dot{H}}{H^2} + \frac{\ddot{H}}{H^3} \text{ and } s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} . \tag{48}$$

In our model the parameters $\{r, s\}$ can be explicitly written as

$$r = \left(2m^2 - 3m + 1\right), \text{ and} \tag{49}$$

$$s = \frac{\left(2m^2 - 3m + 1) - 1\right)}{\frac{3}{2}\left(2q - 1\right)}, \tag{50}$$

and the relation between r and s is

$$2r = (3s-2)(3s-1) . \tag{51}$$

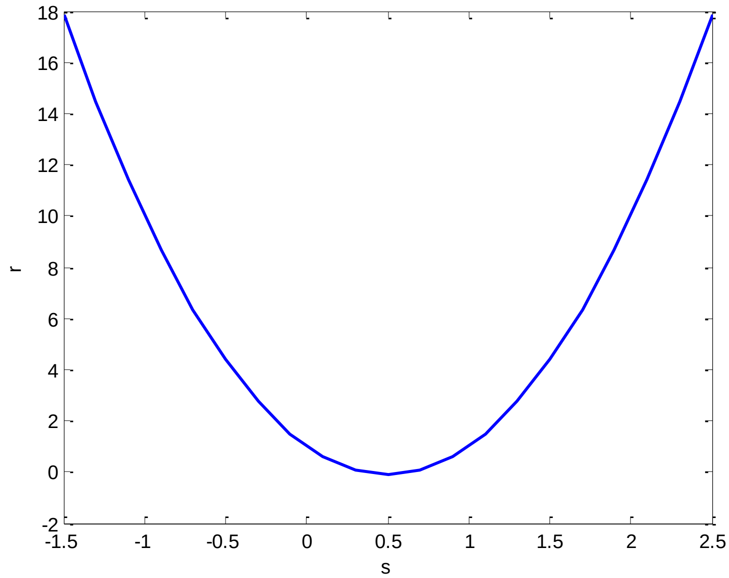

Figure 7. *r* vs *s*.

The universe starts from an asymptotic Einstein static era $(r \to \infty\ ,\ s \to -\infty)$ and goes to ΛCDM model $(r = 1\ ,\ s = 0)$ as shown in figure 7 and which resembles with G.C. Samanta (2013).The physical behavior of the parameters resembles with the behavior of Sarkar (2014).

6. Conclusions

Here we have discussed hypersurface-homogenous space-time in the presence of anisotropic fluid in $f(R,T)$ gravity formulated by Harko et al. (2011) by modifying general relativity to explain the challenging problem of late time acceleration of the universe. To obtain a determinate solution of the highly non-linear field equations of this theory, we have taken the help of special law of variation for Hubble's parameter proposed by Berman (1983). At $t = 0$ the spatial volume V tend to zero when $t \to 0$ and $V \to \infty$ when $t \to \infty$. Thus the model starts evolving with a big-bang at $t = 0$.

The scalar expansion and shear scalar are also take infinite value at the origin of the universe i.e. $t = 0$ and tend to zero as $t \to \infty$.The behavior of energy density ρ versus time is clearly depicted in Fig. 4. It is observed that EoS parameter, skewness parameters in the model are all functions of time. {r, s} diagram (Fig. 7) shows that the evolution of the universe starts from asymptotic Einstein static era $r \to \infty, s \to -\infty$ and approaches to ΛCDM model ($r = 1, s = 0$). We have also discussed the well-known astrophysical phenomena, namely, look-back time, proper distance, luminosity distance, angular diameter distance. It is interesting to note that the results obtained resembles with the investigations of Reddy et. al. (2013).

References

Abazajian, et al.: Phys. Rev. D 69, 103501 (2004)
Adhav, K.S.: Astrophys. Space Sci. 339, 365 (2012)
Ahmed, N., Pradhan, A.: Int. J. Theor. Phys. 53, 289 (2014)
Anil Kumar Yadav,P. K. Srivastava,Lallan Yadav: Int J Theor Phys(2014).DOI 10.1007/s10773-014-2368-2.
Anirudh Pradhan,Rekha Jaiswal, Kanti Jotania,Rajeev Kumar Khare: Astrophys Space Sci (2012) 337:401–413.
B. Mishra, P. K. Sahoo: Astrophysics and Space Science, 352, (2014), 331-336.
Bennett, C.L., et al.: Astrophys. J. Suppl. 148, 1 (2003)
Berman :Nuvo Cimento B, 74,183(1983).
Blandford, R.D., Amin, M., Baltz, E.A., Mandel, K., Marshall, P.L.:arXiv:astro-ph/0408279 (2004)
Chaubey, R., Shukla, A.K.: Astrophys. Space Sci. 343, 415 (2013)
Chiba, T., Nakamura, T.: Prog. Theor. Phys. 100, 1077 (1998).
Collins, C.B., Glass, E.N., Wilkinson, D.A.: Gen. Relativ. Gravit. 12,805 (1980).
D.R.K. Reddy , R.L. Naidu , K. Dasu Naidu ,T. Ram Prasad: Astrophys Space Sci (2013) 346:219–223.
D.R.K. Reddy and R. Santhi Kumar: Astrophys Space Sci (2013) 344:253–257.
D.R.K. Reddy, R.L. Naidu, K. Dasu Naidu,T. Ram Prasad: Astrophys Space Sci (2013) 346:261–265.DOI 10.1007/s10509-013-1426-1.
G.C. Samanta : Int J Theor Phys (2013).DOI 10.1007/s10773-013-1989-1.
G.C. Samanta: Int J Theor Phys (2013) DOI 10.1007/s10773-013-1513-7.
G.C. Samanta: Int J Theor Phys (2013) DOI 10.1007/s10773-013-1556-9.
Hajj-Boutros, J.: J. Math. Phys. 26, 2297 (1985)
Harko, T., et al.: arXiv:1104.2669 [gr-qc] (2011).
Hawkins, E., et al.: Mon. Not. Roy. Astr. Soc. 346, 78 (2003)
Huang, Z.G., Lu, H.Q.: Chin. Phys. Lett. 25, 2732 (2008)
J.K. Singh · N.K. Sharma: Int J Theor Phys (2014) 53:1424–1433.
Jamil, M., Debnath, U.: Astrophys. Space Sci. 333, 3 (2011)
Li, X.Z., Sun, C.B., Xi, P.: J. Cosmol. Astropart. Phys. 0904, 015(2009)
Liu, D.J., Liu, W.Z.: Phys. Rev. D 77, 027301 (2008)
M. Kiran and D.R.K. Reddy: Astrophys Space Sci (2013) DOI 10.1007/s10509-013-1459-5.
M. Sharif and Z. Yousaf: Astrophys Space Sci (2014).DOI 10.1007/s10509-014-2113-6.
P. K. Sahoo, B. Mishra, G Chakradhar Reddy: The Eur. Phys. Jour. Plus, 129, (2014), 49.
P. K. Sahoo, B. Mishra: Canadian Journal of Physics, 92(9), (2014), 1062-1067.
P.K. Sahoo, M. Sivakumar: Astrophysics and Space Science, 357(1):60, (2015), 1-12
Perlmutter, S., et al.: Astrophys. J. 483, 565 (1997)
Perlmutter, S., et al.: Astrophys. J. 517, 565 (1999)
Perlmutter, S., et al.: Nature 391, 51 (1998)
R. Chaubey and A.K. Shukla: Astrophys Space Sci (2012) DOI 10.1007/s10509-012-1204-5.
Reddy, D.R.K., et al.: Int. J. Theor. Phys. 52, 239 (2013)
Reddy, D.R.K., Shanthikumar, R.: Glob. J. Sci. Frontier Res. A 13(2013).
Riess, A.G., et al.: Astron. J. 116, 1009 (1998)
S. D. Katore and A. Y. Shaikh: The African Review of Physics (2014a) 9:0035.
S. Katore, A. Shaikh, Prespacetime J., 3(11), 1087, 2012.
S.D. Katore · K.S. Adhav · A.Y. Shaikh · M.M. Sancheti: Astrophys Space Sci (2011) 333: 333–341.

S.D. Katore · K.S. Adhav · A.Y. Shaikh · N.K. Sarkate: Int J Theor Phys (2010) 49: 2358–2363.
S.D. Katore, A.Y. Shaikh: Astrophys Space Sci (2015) 357:27.
S.D. Katore, A.Y. Shaikh: Bulg. J. Phys. 41 (2014b) 274–290.
S.D. Katore, A.Y. Shaikh: Bulg. J. Phys. 42 (2), 29–41(2015)
S.D. Katore, A.Y. Shaikh: Prespacetime Journal, March 2015 , Volume 6 , Issue 3 , pp. 207-217
S.D. Katore, A.Y. Shaikh: Rom. Journ. Phys., Vol. 59, Nos. 7–8, P. 715–723, Bucharest, (2014c).
Sahni, V., Saini, T.D., Starobinsky, A.A., Alam, U.: JETP Lett. 77, 201(2003)
Sahni, V.: arXiv:astro-ph/0211084 (2002).
Sanjay Sarkar : Astrophys Space Sci (2014) 351:361–369.
Sanjay Sarkar: Astrophys Space Sci (2014) 349:985–993.
Sarita Rani ,J. K. Singh, N. K. Sharma:Int J Theor Phys (2014).DOI10.1007/s10773-014-2371-7.
SD Katore, AY Shaikh: The African Review of Physics 7 (0004)(2012).
SD Katore, AY Shaikh:International Journal of Theoretical Physics 51 (6), 1881-1888(2012).
Shamir, M.F.: J. Exp. Theor. Phy. 119(2), 242–250(2014)
Sharif, M., Zubair, M.: Astrophys. Space Sci. 349, 457 (2014).
Sharma, N.K., Singh, J.K.: Int. J. Theor. Phys. (2014). doi:10.1007/s10773-014-2089-6.
Shri Ram and M.K. Verma: Astrophys Space Sci (2010)DOI 10.1007/s10509-010-0370-6.
Spergel, D.N., et al.: Astrophys. J. Suppl. 148, 175 (2003)
Stewart, J.M., Ellis, G.F.R.: J. Math. Phys. 9, 1072 (1968)
V. R. Chirde and S. H. Shekh: Astrophysics, Vol. 58, No. 1, March , 2015.
V.U.M. Rao and D. Neelima: Astrophys Space Sci (2013) 345:427–430.
V.U.M. Rao and D. Neelima: Eur. Phys. J. Plus (2013) 128: 35.
V.U.M. Rao and D.C. Papa Rao: Astrophys Space Sci (2015) 357:65.
V.U.M. Rao and D.C. Papa Rao: Astrophys Space Sci (2015) 357:77.DOI 10.1007/s10509-015-2242-6.
Verde, L., et al.: Mon. Not. R. Astron. Soc. 335, 432 (2002)
Visser, M.: Class. Quantum Gravity 21, 2603 (2004)
Visser, M.: Gen. Relativ. Gravit. 37, 1541 (2005)