Article

Tilted Kasner-Type Cosmological Models in Brans-Dicke Theory of Gravity

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Abstract

Tilted Kasner type metric is considered in the presence of Brans-Dicke theory of gravity. We find that it is not possible to describe an anisotropic physical model of the Universe.

Keywords: Tilted model, Brans-Dicke theory, Kasner type-I model.

1. Introduction

There have been numerous investigations of spatially homogeneous (SH) cosmological models with a perfect fluid as a matter source. In the study of these models one usually distinguish between two different scenarios: when the fluid congruence is normal to the hypersurfaces of homogeneity and when it is not. The models of the first kind are called orthogonal or non-tilted, and the second kinds are called tilted models. King and Ellis [1]; Ellis and King [2]; Collins and Ellis [3] have examined the general dynamics of tilted cosmological models. Tilted Bianchi type-I cosmological model for perfect fluid have been investigated by Dunn and Tupper [4]. Lorentz [5] has obtained Tilted electromagnetic Bianchi type-I cosmological model in General Relativity. Beesham [6] has presented tilted Bianchi type V cosmological models in the scale- covariant theory. Different aspects of tilted cosmological models obtained by Hewitt et al. [7,8], Horwood et al [9] , Apostolopoulos [10].

Bali and Sharma [11] have studied Tilted Bianchi type-I dust fluid. Bali and Meena [12] have investigated Tilted cosmological models filled with disordered radiation in General Relativity. Pradhan and Rai [13] have presented Tilted Bianchi type I cosmological models filled with disordered radiation in general relativity. Pawar et al. [14] discussed tilted plane symmetric cosmological models with heat conduction and disordered radiation. Conformally flat tilted cosmological models studied by Pawar and Dagwal [15]. Tilted plane symmetric bulk viscous cosmological model with varying Λ–Term have studied by Bhaware et al. [16]. Tilted Bianchi type-I cosmological model in Lyra Geometry derived by Sahu and Kumar [17]. Bagora and Purohit [18] discussed tilted Bianchi type-I barotropic cosmological model and some Bianchi

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type-I magnetized bulk viscous fluid tilted cosmological models. Recently two fluids tilted cosmological model in General Relativity and Kantowski-Sachs cosmological model in scalar tensor theory of gravitation presented by Pawar and Dagwal [19-20]. Pawar et al. studied tilted plane symmetric magnetized cosmological models [21].

Brans and Dicke [22] have studied Mach's principle in a relativistic framework by assuming the interaction of inertial masses of fundamental particles with some cosmic scalar field, coupled with the large scale distribution of matter in motion, has gained momentum. Reddy and Rao [23] have examined field of a charged particle in Brans-Dicke theory of gravitation. Shri Ram and Singh [24] have presented Exact Bianchi type VI0 cosmological solutions with matter in Brans-Dicke theory. Berman et. al. [25] has discussed Brans-Dicke static universes. Pawar et al. [26] have obtained cosmological models with thick domain walls in Brans Dicke theory of Gravitation. Rao and Santhi [27] have discussed Bianchi type-II, VIII & IX perfect fluid magnetized cosmological models in Brans-Dicke theory of gravitation. Recently Pawar and Solanke [28] have investigated the Kantowski-Sachs dark energy cosmological model in Brans-Dicke theory of gravitation.

The Kasner [29] universe refers to a vacuum cosmological model in Einstein's theory. An analytic expression for the anisotropy of the Kasner metric has studied by Gron [30,31]. Bianchi type-I cosmological model with gauge function in vacuum where in they have shown that the model reduces to Kasner form when cosmological constant is zero and the model isotropizes in Einstein's theory for a non zero cosmological constant have examined by Mohanty and Daud [32]. Viscous cosmological fluid does not permit the Kasner metric to be anisotropic in Einstein's general relativity proved by Barros and Romer [33], Brevik and Petterson [34]. Cataldo and del Campo [35] have investigated in general relativity the Bianchi type I metric of the Kasner form is not able to describe an anisotropic universe filled with a viscous fluid, satisfying simultaneously the dominant energy condition (DEC) and the second law of thermodynamics. String cosmology in in Brans-Dicke theory for Kasner type metrics discussed by Adhav et al. [36]. Interacting Kasner-type cosmology's presented by Cataldo et al. [37].

2. Field Equation

We consider the metric in the form

$$ds^2 = -dt^2 + t^{2p_1}dx^2 + t^{2p_2}dy^2 + t^{2p_3}dz^2 , \tag{1}$$

where $p_{1,}\, p_2$ *and* p_3 are Kasner constant parameter .

The Einstein's field equations in Brans- Dicke for the combined scalar and tensor field are given by

$$R_i^j - \frac{1}{2} g_i^j R = -8\pi\phi^{-1}\left(-T_i^j\right) - \omega\phi^{-2}(\phi_{,i}\phi^{,j} - 1/2\, g_i^j \phi_{,k}\phi^{,k}) - \phi^{-1}\left(\phi_{;i}^{j} - \phi_{;k}^{,k}\right). \tag{2}$$

where

$$\phi^{;k}_{;k} = \Box\phi = \frac{1}{\sqrt{-g}}\left[\sqrt{-g}\,\phi^{k}\right]_{,k} = \frac{8\pi\phi^{-1}T}{(3+2\omega)}, \tag{3}$$

ϕ is a Brans- Dicke scalar field and ω is the dimensionless coupling constant.

The energy momentum tensor for perfect fluids given by

$$T_i^{\ j} = (\rho + p)u_i u^j + p\, g_i^j + q_i u^j + u_i q^j \quad , \tag{4}$$

together with

$$g_{ij}u^i u^j = -1 \quad , \tag{5}$$

$$q_i q^i > 0, and \quad q_i u^i = 0 . \tag{6}$$

here p is the pressure, ρ is the energy density, q_i is the heat conduction vector orthogonal to u^i.

The fluid vector u^i has the components $\left(\frac{\sinh\alpha}{t^{p_1}}, 0, 0, \cosh\alpha\right)$ satisfying Equation (5) and α is the tilt angle.

The Einstein's field equation (2) reduces to

$$\left[p_1(s-1) - \tfrac{1}{2}\left(s^2 - 2s + Q\right)\right]t^{-2} = -8\pi\phi^{-1}\left[(\rho + P)\sinh^2\alpha + p + 2q_1\frac{\sinh\alpha}{t^{p_1}}\right] + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_2}{t} + \frac{p_3}{t}\right) + \frac{\phi_{44}}{\phi} \quad , \tag{7}$$

$$\left[p_2(s-1) - \tfrac{1}{2}\left(s^2 - 2s + Q\right)\right]t^{-2} = -8\pi\phi^{-1}p + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_1}{t} + \frac{p_3}{t}\right) + \frac{\phi_{44}}{\phi} \quad , \tag{8}$$

$$\left[p_3(s-1) - \tfrac{1}{2}\left(s^2 - 2s + Q\right)\right]t^{-2} = -8\pi\phi^{-1}p + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_1}{t} + \frac{p_2}{t}\right) + \frac{\phi_{44}}{\phi} \,, \tag{9}$$

$$\left[\tfrac{1}{2}\left(Q - s^2\right)\right]t^{-2} = 8\pi\phi^{-1}\left[(\rho + P)\cosh^2\alpha - p + 2q_1\frac{\sinh\alpha}{t^{p_1}}\right] - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_1}{t} + \frac{p_2}{t} + \frac{p_3}{t}\right), \tag{10}$$

$$-8\pi\phi^{-1}\left[\left(\rho+P\right)t^{p_1}\sinh\alpha\cosh\alpha+q_1\cosh\alpha+q_1\frac{\sinh^2\alpha}{\cosh\alpha}\right]+\frac{\phi_4}{\phi}\left(\frac{p_1}{t}+\frac{p_2}{t}+\frac{p_3}{t}\right)+\frac{\phi_{44}}{\phi}=0 , \quad (11)$$

$$\phi_{44}+\phi_4\left(\frac{p_1}{t}+\frac{p_2}{t}+\frac{p_3}{t}\right)=-\frac{8\pi\phi^{-1}}{(3+2\omega)}(3p-\rho) . \quad (12)$$

where the suffix 4 after the field denotes ordinary differentiation with respect to time.

Comparing Equation (8) and (9) we get

$$\frac{s-1}{t}=-\frac{\phi_4}{\phi} . \quad (13)$$

Integrate Eq. (13) we give

$$\phi=\frac{c}{t^{(s-1)}} , \quad (14)$$

where c is integration constant .

Using Equation (14) and s =1 we get

$$\phi_4=0 . \quad (15)$$

Equation (8), (12) and (15) for s=1 we get

$$p=\frac{(Q-1)c}{16\pi t^2} , \quad (16)$$

$$\rho=\frac{3(Q-1)c}{16\pi t^2} . \quad (17)$$

Equation (7),(10) and (11) we get tilt angle α and heat conduction q_1 & q_4

$$\cosh\alpha=\sqrt{3/2} \;\;\&\;\; \sinh\alpha=\sqrt{1/2} , \quad (18)$$

$$q_1=\frac{3c\sqrt{1/2}(1-Q)t^{p_1-2}}{16\pi} \;\;\&\;\; q_4=\frac{c\sqrt{3/2}(Q-1)t^2}{16\pi} \quad (19)$$

Relation between parameter are given by

$$s=p_1+p_2+p_3=1 \quad and \quad Q=p_1^2+p_2^2+p_3^2=1 \quad (20)$$

Using Equation (20) we get Equation (7) - (12), (15) - (17) and (19) are

$$p=\rho=\phi_{4}=q_1=q_4=0. \tag{21}$$

The pressure, energy density, Brans- Dicke scalar field and the heat conduction vector of the fluid distribution do not survive. Thus in this model the space time becomes Minkowskian.

3. Conclusion

It is not possible to describe an anisotropic physical model of the Universe in Tilted Kasner type-I cosmological models in Brans-Dicke theory of gravity. Tilt angles α are constant. This conclusion match with the conclusions presented by Cataldo and del Campo [35], Adhav et al. [36].

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