Article

Expanding & Shearing Biachi Type-I Non-Static Cosmological Model in General Relativity

Reena Mathur[1], Gajendra P. Singh[*2] & Atul Tyagi[1]

[1]Department of Mathematics and Statistics, MohanLal Sukhadia University, Udaipur
[2]Department of Mathematics, Geetanjali Institute of Technical Studies, Udaipur

Abstract

We have investigated expanding, shearing and non- static Bianchi type-I cosmological models in General Relativity. To get deterministic model, we have assumed that shear (σ) is proportional to the expansion (θ) in the model which leads to $A= (BC)^n$ where A, B, C are metric potential and n is constant. Using the condition, we found a stiff fluid model. The various physical and geometrical aspects of model are also discussed.

Key Words: Expanding, shearing, electromagnetic field, cosmological Model.

Introduction

The cosmological model in presence of magnetic field, play a significant role in the evolution of galaxies and stellar bodies. Primordial magnetic of cosmological origin is speculated by Asseo and Sol [1]. The break down of isotropy is also due to the magnetic field. Collins [2] gave a qualitative analysis of Bianchi Type I model with magnetic field. Jacobs [3] investigated Bianchi Type I cosmological model with barotropic fluid in presence of magnetic field. Bali and Tyagi [4] have investigated magnetized Bianchi type I orthogonal cosmological model for perfect fluid distribution in General Relativity. Bali and Meena [5] have investigated magnetized stiff fluid tilted universe for perfect fluid distribution in General Relativity.

The presence of magnetic field in galactic and intergalactic spaces is evident from recent observations [6]. The large scale magnetic fields can detect by observing their effects on the CMB radiation. These fields would enhance anisotropies in the CMB, since the expansion rate will be different depending on the direction of the field lines [7, 8]. Matravers and Tsagas [9] found that interaction of cosmological magnetic field with the space time geometry could affect the expansion of the universe. If the curvature is strong, then even the weak magnetic field will affect the evolution of the universe. The magneto-curvature coupling tends to accelerate the positively curved regions while it decelerates the negatively curved regions [10].

Jacobs [11] studied the spatially homogeneous and anisotropic Bianchi type I cosmological model with expansion and shear but without rotation. He discussed anisotropy in the temperature of CMB and expansion both with and without magnetic field. It was concluded that the primordial magnetic field produce large expansion anisotropies during radiation-dominated

[*]Correspondence: Gajendra P. Singh, Department of Mathematics, Geetanjali Institute of Technical Studies, Udaipur.
E-mail: gajendrasingh237@gmail.com

phase but it has negligible effect during the dust dominated phase. Dunn and Tupper [12] discussed properties of Bianchi type VI_0 models with perfect fluid and magnetic field. Roy *et.al* [13] explored the effects of cosmological constant in Bianchi type I and VI_0 models with perfect fluid and homogeneous magnetic field in axial direction.

Several researchers like Zeldovich [14], Bertolmi [15], Ozer and Taha [16], Friemann and Waga [17] and Pradhan *et.al* [18] investigated more significant cosmological model with cosmological constant. Bianchi type III cosmological models are studied by numbers of researcher in different context viz. Tikekar and Patel [19], Bali and Dave [20], Bali and Pradhan [21]. Singh and Tyagi [22-24] investigated various Bianchi Type cosmological models with variable cosmological and gravitational constant in presence and absence of magnetic field. The perfect and bulk viscous fluids are considered as source of matter. Singh et.al. [25] have investigated LRS Bianchi type III Massive String cosmological model with Electromagnetic field.

In this paper, we have investigated expanding, shearing and non- static Bianchi type-I cosmological models in General Relativity. To get deterministic model, we have assumed that shear (σ) is proportional to the expansion (θ) in the model which leads to $A= (BC)^n$ where A, B, C are metric potential and n is constant. It is observe that the model leads to stiff fluid case in general. The physical and geometrical implications of models are also discussed.

The Metric & Field Equations

We consider homogeneous anisotropic Bianchi type-I metric in the form of

$$ds^2 = A^2\left(dx^2 - dt^2\right) + B^2dy^2 + C^2dz^2 \tag{1}$$

where A, B and C are function of t alone.

In this paper we have considered distribution of matter is consist of perfect fluid with an infinite electrical conductivity and a magnetic field.

The energy-momentum tensor of the composite field is assumed to be the sum of the corresponding energy-momentum tensor. Thus

$$T_i^j = (p+\rho)v_i v^j + pg_i^j + E_i^j \tag{2}$$

Were ρ is energy density, p is effective pressure and v^j the flow vector satisfying the relation

$$g_{ij}v^i v^j = -1 \tag{3}$$

Also, E_i^j is the electromagnetic field given by Lichnerowicz [26] as

$$E_i^j = \mu\left[|h|^2\left(v_i v^j + \frac{1}{2}g_i^j\right) - h_i h^j\right] \tag{4}$$

here μ is the magnetic permeability and h_i is the magnetic flux vector define by

$$h_i = \frac{1}{\mu}{}^*F_{ji}v^j \tag{5}$$

where $^*F_{ij}$ is the dual electromagnetic field tensor define by Synge [27] as

$$^{*}F_{ji} = \frac{1}{2}\sqrt{-g}\,\varepsilon_{jikl}F^{kl} \tag{6}$$

F_{ij} is electromagnetic field tensor and ε_{ijkl} levi-cevita tensor density. Here, the co-moving coordinates are taken to be $v^1 = v^2 = v^3 = 0$ *and* $v^4 = \frac{1}{A}$.We take the incident magnetic field to be in the direction of the *x*-axis so that $h_1 \neq 0$, $h_2 = 0 = h_3 = h_4$. On the assumption of infinite conductivity of the fluid, we get $F_{14} = 0 = F_{24} = F_{34}$ .The only non –vanishing component of F_{ij} is $F_{23.}$

The first set of Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \;\; and \;\; F^{ij}_{;k} = 0$$

are satisfied by

$$F_{23} = H \text{ (constant)} \tag{7}$$

where the semicolon represent a covariant differentiation.

From equation (5) and (7) we find that

$$h_1 = \frac{AH}{\mu BC} \tag{8}$$

The Einstein's field equation ($G = c = 1$) read as

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi T_i^j \tag{9}$$

Einstein field equation (9) together with (2) and (4) for metric (1) lead to

$$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{B_4C_4}{BC} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC}\right] = 8\pi\left(p - \frac{H^2}{2\mu B^2C^2}\right) \tag{10}$$

$$\frac{1}{A^2}\left[-\frac{A_{44}}{A} - \frac{C_{44}}{C} + \frac{A_4^{\,2}}{A^2}\right] = 8\pi\left(p + \frac{H^2}{2\mu B^2C^2}\right) \tag{11}$$

$$\frac{1}{A^2}\left[-\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{A_4^{\,2}}{A^2}\right] = 8\pi\left(p + \frac{H^2}{2\mu B^2C^2}\right) \tag{12}$$

$$\frac{1}{A^2}\left[\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC}\right] = 8\pi\left(\rho + \frac{H^2}{2\mu B^2C^2}\right) \tag{13}$$

Solutions of the Field Equations

Equations (10), (11) and (12) leads to

$$\left(\frac{A_4}{A}\right)_4+\frac{A_4}{A}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)=\frac{B_{44}}{B}+\frac{B_4C_4}{BC}-\frac{8\pi H^2 A^2}{\mu B^2C^2} \tag{14}$$

And

$$\frac{B_{44}}{B}=\frac{C_{44}}{C} \tag{15}$$

Equations (10) to (13) represent a system of four equations in five unknowns A, B, C, p and ρ. To get determinate model we use an extra condition between metric potentials as

$$A=(BC)^n \tag{16}$$

equations (14) and (16), lead to

$$(B_4\,C)_4=\frac{K^2 A^2}{BC},\ \text{Where}\ \ K^2=\frac{8\pi H^2}{(1-2n)\mu} \tag{17}$$

Putting

$$\mu = BC \ \text{and}\ \nu=\frac{B}{C} \tag{18}$$

Equations (15) and (17) lead to

$$\mu_{44}+\left(\frac{\nu_4\,\mu}{\nu}\right)_4-\frac{2K^2A^2}{\mu}=0 \tag{19}$$

And

$$\left(\frac{\nu_4\,\mu}{\nu}\right)_4=0 \tag{20}$$

Equation (20) leads to

$$\frac{\nu_4}{\nu}=\frac{k}{\mu} \tag{21}$$

where *k* is constant of integration.

Using equation (19) and (20), we get

$$\mu\,\mu_{44}=2K^2A^2 \tag{22}$$

Equation (22) leads to

$$\frac{\mu_4}{2\mu}+\frac{\mu_{444}}{2\mu_{44}}=\frac{A_4}{A} \tag{23}$$

Equation (16), (18) and (22) lead to

$$B_4 C = K^2 a \mu_4 + L \tag{24}$$

where L is constant of integration.

Equation (18), (21) and (24) leads to

$$k(1 - K^2 a) - L = (2K^2 a - 1) B C_4 \tag{25}$$

Equations (18) and (25) leads to

$$\frac{k - 2L}{2K^2 a - 1} = \mu_4 \tag{26}$$

which leads to

$$\mu = (\alpha t + N) \tag{27}$$

where $\alpha = \dfrac{k - 2L}{2K^2 a - 1}$

And

$$\nu = \beta (\alpha t + N)^{k/\alpha} \tag{28}$$

where β and N are constant of Integration.

From equations (16), (18), (27) and (28) we get

$$A = (\alpha t + N)^n \tag{29}$$

$$B = \sqrt{\mu \nu} = \sqrt{\beta}\, (\alpha t + N)^{\frac{k+\alpha}{2\alpha}} \tag{30}$$

$$C = \sqrt{\mu / \nu} = \frac{1}{\sqrt{\beta}} (\alpha t + N)^{\frac{\alpha - k}{2\alpha}} \tag{31}$$

Hence, the metric reduces to the form

$$ds^2 = (\alpha t + N)^{2n} (dx^2 - dt^2) + \beta (\alpha t + N)^{\frac{k+\alpha}{\alpha}} dY^2 + \frac{1}{\beta} (\alpha t + N)^{\frac{\alpha - k}{\alpha}} dZ^2 \tag{32}$$

After suitable transformation of co-ordinates, the metric reduces to the form

$$ds^2 = T^{2n} \left(dX^2 - \frac{dT^2}{\alpha^2} \right) + T^{\frac{k+\alpha}{\alpha}} dY^2 + T^{\frac{\alpha - k}{\alpha}} dZ^2 \tag{33}$$

where $(\alpha t + N) = T, x = X, y = Y, z = Z$.

Some Physical & Geometrical Features

The pressure and density for the model (33) are given by

$$8\pi p = \frac{-4\pi H^2}{\mu T^2} + \frac{1}{T^{2(n+1)}}\left(\frac{(4n+1)\alpha^2 - k^2}{4}\right) \tag{34}$$

$$8\pi \rho = \frac{-4\pi H^2}{\mu T^2} + \frac{1}{T^{2(n+1)}}\left(\frac{(4n+1)\alpha^2 - k^2}{4}\right) \tag{35}$$

The model has to satisfy the reality conditions given by Ellis [28]

(i) $\rho + p > 0$ (ii) $\rho + 3p > 0$

Leads to

$$T^{2n} < \left(\frac{(4n+1)\alpha^2 - k^2}{16\pi H^2}\right)\mu \tag{36}$$

The scalar of expansion θ calculated for the flow vector v^i is given by

$$\theta = \frac{(n+1)\alpha}{T^{n+1}} \tag{37}$$

The scalar of shear is given by

$$\sigma^2 = \frac{1}{18T^{2(n+1)}}\left[\alpha^2(2n-1)^2 + \frac{(3k+\alpha-2n\alpha)^2}{4} + \frac{(\alpha-2n\alpha-3k)^2}{4\beta^{3/2}}\right] \tag{38}$$

Now,

$$\frac{\sigma}{\theta} = \frac{1}{3(n+1)\alpha}\left[\frac{1}{2}\left\{\alpha^2(2n-1)^2 + \frac{(3k+\alpha-2n\alpha)^2}{4} + \frac{(\alpha-2n\alpha-3k)^2}{4\beta^{3/2}}\right\}\right]^{1/2} \tag{39}$$

The non vanishing components of shear tensor σ_{ij} are given by

$$\sigma_{11} = \frac{\alpha(2n-1)}{3}T^{(n-1)} \tag{40}$$

$$\sigma_{22} = \frac{\beta}{6}[3k + \alpha(1-2n)]T^{\frac{k-n\alpha}{\alpha}} \tag{41}$$

$$\sigma_{33} = \frac{1}{6\beta}[\alpha(1-2n) - 3k]T^{-n-\frac{k}{\alpha}} \tag{42}$$

The rotation ω is identically zero and the non vanishing components of conformal curvature tensor are given by

$$C_{12}^{12} = C_{34}^{34} = \frac{1}{12T^{2n+2}}\left[\alpha^2(1-2n) - k^2 - 6n\alpha k\right] \tag{43}$$

$$C_{13}^{13} = C_{24}^{24} = \frac{1}{12T^{2n+2}}\left[\alpha^2(1-2n) - k^2 + 6n\alpha k\right] \tag{44}$$

$$C_{14}^{14} = C_{24}^{24} = \frac{1}{6\,T^{2n+2}}\left[\alpha^2(1-2n) + k^2\right] \tag{45}$$

Conclusion

The models start with big bang at T= 0 for n > -1 and the expansion in the model decreases as time increases. The expansion stops at n = - 1 or T $\to\infty$. The model represents an expanding, shearing and non rotating universe. As T$\to\infty$, p $\to$0 and $\rho\to$0 provided n + 1 > 0. Since T$\to\infty$ $\frac{\sigma}{\theta} \neq 0$ .Hence the model doesn't approach isotropy for large value of T. A point type singularity [29] is observed for $n > 0$ and k > α as T$\to$0, $g_{11}\to 0$, $g_{22}\to 0$, $g_{33}\to 0$.

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