Article

Bianchi Type VIII Inflationary Universe with Massless Scalar Field in General Relativity

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Abstract

Bianchi Type VIII inflationary universe in the presence of massless scalar field with flat potential is investigated. To get the deterministic model of the universe, we assume that the shear (σ) is proportional to expansion (θ) as considered by Thorne [20] and Collins et al.[21]. The model represents decelerating and accelerating phases of universe and spatial volume increases exponentially with time representing inflationary scenario. The model initially represents anisotropic space-time. However, it isotropizes in special case. The Higgs field evolves slowly but the universe expands.

Keywords: Bianchi VIII, inflationary, massless scalar field.

1. Introduction

Anisotropic cosmological models including the so called Bianchi cosmologies (Ellis et al.[1], Gron and Hervik[2]) are of great theoretical importance and have been object of close studies (Hewitt et al.[3]), Coley and Hervik[4]). Bianchi Type VIII cosmological models are the most general ever-expanding Bianchi cosmologies and are therefore of special interest. The late time behaviour of Bianchi Type VIII space-time in presence of diffusion is studied by Shogin and Hervik[5].

The state of accelerated expansion of the universe is termed as Inflation. It was first proposed in the beginning of the 1980s and now a days receives a great deal of attention. The inflationary epoch comprises the first part of the electro weak epoch following the grand unification epoch. It lasted from 10^{-33} to 10^{-32} seconds. Following the inflationary period, the universe continues to expand. Inflationary universes provide a potential solution to the formation of structure problem in Big-Bang cosmology like Horizon, Flatness and Magnetic monopole problems. Guth[6] introduced the idea of early inflationary phase in the context of grand unified field theories (Zel'dovich and Khlopov[7]) in which symmetry breaking phase transitions occur with the decrease of temperature at the very early stages of evolution of universe (Linde[8]). Rothman and Ellis[9] explained that we can have isotropic space-time if we work with anisotropic metrics and these metrics can be isotropized in a very general circumstances. Stein-Schabes[10] has shown that inflation will take place if effective potential $V(\phi)$ has flat region when Higgs field (ϕ) evolves slowly but the universe expands in an exponential way due to vacuum field energy. It has been assumed that the scalar field takes sufficient time to cross the flat region so that

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universe expands slowly to become homogeneous and isotropic. Anninos et al.[11] has discussed the significance of inflation for isotropization of the universe. Many authors viz. Bali et al.[12,13,14,15], Singh and Kumar[16], Reddy et al.[17,18], Katore et al.[19] studied inflationary cosmological models in different Bianchi space-times with self interacting scalar field in General Relativity.

Motivated by the above mentioned studies, we investigate inflationary scenario in Bianchi Type VIII space-time for a massless scalar field with flat potential using the condition that shear (σ) is proportional to expansion (θ). We find that the model represents accelerating universe and spatial volume increases exponentially representing inflationary scenario. The model represents anisotropic space-time initially but isotropizes in special case.

2. Metric and Field Equations

We consider Bianchi Type VIII metric in the form

$$ds^2 = -dt^2 + R^2(t)[d\theta^2 + \cosh^2\theta d\phi^2] + S^2(t)[d\psi + \sinh\theta\, d\phi]^2 \tag{1}$$

where R and S are functions of t-alone. The Lagrangian is that of gravity minimally coupled to Higgs scalar field (ϕ) with effective potential V(ϕ) given by Stein-Schabes[10]:

$$L = \int \sqrt{-g}\left[R - \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right]d^4x \tag{2}$$

The variation of L with respect to the dynamical field, leads to the Einstein field equation

$$R_i^j - \frac{1}{2}R\,g_i^j = -T_i^j \tag{3}$$

(in geometrical unit $8\pi G = 1$, $c = 1$) where

$$T_{ij} = \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)\right]g_{ij} \tag{4}$$

and

$$\frac{1}{\sqrt{-g}}\partial_i[\sqrt{-g}\,\partial_i\phi] = -\frac{dV}{d\phi} \tag{5}$$

where ϕ is the Higgs field, V the effective potential and g_{ij} the metric tensor.

Einstein's field equation (3) with (4) and (5) for the metric (1) leads to

$$\frac{\dot{R}\dot{S}}{RS} + \frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{S^2}{4R^4} = -\left[\frac{\dot{\phi}^2}{2} - V(\phi)\right] \tag{6}$$

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{3S^2}{4R^4} - \frac{1}{R^2} = -\left[\frac{\dot{\phi}^2}{2} - V(\phi)\right] \tag{7}$$

$$\frac{\dot{R}^2}{R^2} + \frac{2\dot{R}\dot{S}}{RS} - \frac{1}{R^2} - \frac{S^2}{4R^4} = \frac{\dot{\phi}^2}{2} + V(\phi) \tag{8}$$

The equation for scalar field leads to

$$\ddot{\phi}+\left(\frac{2\dot{R}}{R}+\frac{\dot{S}}{S}\right)\dot{\phi}=\frac{dV}{d\phi} \tag{9}$$

To get the deterministic solution in terms of cosmic time t, we assume that shear (σ) is proportional to expansion (θ) as considered by Thorne[20] and Collins et al.[21]. Thus, we have

$$S=R^{n} \tag{10}$$

As considered before[13], we consider that effective potential $V(\phi)$ is flat and $V(\phi)$ = constant to get the inflationary scenario. Thus equation (9) leads to

$$\ddot{\phi}+\left(\frac{2\dot{R}}{R}+\frac{\dot{S}}{S}\right)\dot{\phi}=0 \tag{11}$$

where

$$V(\phi) = k \text{ (constant)}. \tag{12}$$

Equations (7) and (8) lead to

$$\frac{2\ddot{R}}{R}+\frac{2\dot{R}^{2}}{R^{2}}+\frac{2\dot{R}\dot{S}}{RS}-\frac{2}{R^{2}}-\frac{S^{2}}{R^{4}}=2k \tag{13}$$

From equations (13) and (10), we have

$$2\ddot{R}+\frac{(2n+2)\dot{R}^{2}}{R}=\frac{2kR^{2}+R^{2n-2}+2}{R} \tag{14}$$

To get the solution, we assume that $\dot{R}=f(R)$. Thus equation (14) leads to

$$2f\,f^{'}+\frac{2n+2}{R}f^{2}=\frac{2kR^{2}+R^{2n-2}+2}{R} \tag{15}$$

which leads to

$$f^{2}=\frac{k}{n+2}R^{2}+\frac{R^{2n-2}}{4n}+\frac{2}{2n+2}+\alpha R^{-2n-2} \tag{16}$$

where

$$f^{'}=\frac{df}{dR} \tag{17}$$

To get the solution in terms of cosmic time t, we assume n = 2 and α = 0. Thus equation (16) leads to

$$\left(\frac{dR}{dt}\right)^{2}=\left(\frac{k}{4}+\frac{1}{8}\right)R^{2}+\frac{1}{3} \tag{18}$$

which leads to

$$R=a\sinh(\gamma t+\delta) \tag{19}$$

where

$$\gamma^{2}=\frac{2k+1}{8},a^{2}=\frac{1}{3\gamma^{2}} \tag{20}$$

Thus

$$S = R^2 = a^2 \sinh^2(\gamma t + \delta) \tag{21}$$

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2 = -\frac{dT^2}{\gamma^2} + a^2 \sinh^2 T\,[d\theta^2 + \cosh^2\theta\, d\phi^2] + a^4 \sinh^4 T[d\psi + \sinh\theta\, d\phi)^2 \tag{22}$$

3. Physical and Geometrical Aspects

Using the assumed condition $V(\phi)$ = constant = k, the equation (11) leads to

$$\dot{\phi} = \frac{\ell}{a^4} \operatorname{cosech}^4 T \tag{23}$$

which leads to

$$\phi = \frac{\ell}{a^4 \gamma}\left[-\frac{\coth^3 T}{3} + \coth T + \frac{N}{3}\right] \tag{24}$$

where ℓ and N are constants. Higgs field (ϕ) decreases slowly and for large value of T, it tends to a finite value. The spatial volume (V^3) is given by

$$V^3 = R^2 S = a^4 \sinh^4 T \tag{25}$$

The directional Hubble parameters H_x, H_y, H_z are given by

$$H_x = H_y = \frac{\dot{R}}{R} = \gamma \coth T \tag{26}$$

$$H_z = \frac{\dot{S}}{S} = \frac{2\dot{R}}{R} = 2\gamma \coth T \tag{27}$$

The mean Hubble parameter (H) is given by

$$H = \frac{1}{3}(H_x + H_y + H_z) = \frac{4\gamma}{3} \coth T \tag{28}$$

The expansion (θ), the shear (σ) and deceleration parameter (q) are given by

$$\theta = 3H = 4\gamma \coth T \tag{29}$$

$$\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{3} H_i^2 - \frac{1}{3}\theta^2\right] = \frac{\gamma^2}{3} \coth^2 T$$

Thus, we have

$$\sigma = \frac{\gamma}{\sqrt{3}} \coth T \tag{30}$$

$$q = -\frac{\ddot{V}/V}{\dot{V}^2 / V^2} = -\left[\frac{\coth^2 T}{4} + \frac{3}{4}\tanh^2 T\right] \tag{31}$$

Discussion and Conclusion

The Higgs field decreases slowly but the universe expands and at late time, it has finite value. The spatial volume increases exponentially representing inflationary scenario. Since $\frac{\sigma}{\theta} \neq 0$, hence the model represents anisotropic space-time in general. However, it represents isotropic space-time for n = 1. The Hubble parameter (H) is initially large but decreases with time. The model describes an unified expansion history of the universe which starts with decelerating expansion and the expansion accelerates at the late time. The decelerating expansion at initial epoch provides obvious provision for the formation of large structure in the universe. The formation of structure is better supported by decelerating expansion. Thus the model is astrophysically relevant. Also late time acceleration is in agreement with the observations of 16 Type Ia supernovae made by Hubble space Telescope (Riess et al.[23]). The model has Point Type singularity at T = 0 (MacCallum[22]).

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