Article

On Principal Invariants in Canonical Coordinate System in V5

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Abstract

In this paper we studied canonical co-ordinate system (Ccs), standard co-ordinate systemfor g_{ij}, special co-ordinate system (Scs) and the principal invariants in Scs in a narrow sense of V_5.

Keywords: Spherically symmetric, principal invariants, narrow sense.

1. Introduction

According to Takeno [2] the space time V_4 with metric

$$ds^2 = g_{ij} dx^i dx^j \tag{1.1}$$

is spherically symmetric if

$$\mathrm{L}_\xi \, g_{ij} = 0 \tag{1.2}$$

where L_ξ denotes the Lie derivative with respect to the Killing vector ξ^i.

Takeno has obtain the most general form of the s.s. line element in spherical polar coordinate (r,θ,ϕ,t) as

$$ds^2 = -A\, dr^2 - B\,(d\theta^2 + \sin^2\theta\, d\varphi^2) + C\, dt^2 + 2D\, dr\, dt \tag{1.3}$$

Takeno has further reduced line element (1.3) in the s.s. coordinate system in a narrow sense

$$ds^2 = 2D\, dr\, dt - B\,(d\theta^2 + \sin^2\theta\, d\varphi^2) \tag{1.4}$$

by the transformation of coordinate *T*.

Karade T.M. and Thomas K.T.[4] have obtained the most general form of the s.s. line element in V_5 as

$$ds^2 = -Adr^2 - B\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right) + Cdt^2 - Ddu^2 + 2Edrdt + 2Fdrdu + 2Gdtdu \tag{1.5}$$

where *A*, *B*, *C*, *D*, *E*, *F* and *G* are the functions of *r*, *t* and *u*, and $x^i \equiv (r,\theta,\phi,t,u)$.

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Further Pokey s.s. and Thomas K.T.[3] has reduced the line element (1.5) into the form

$$ds^2 = -Adr^2 - B\left(d\theta^2 + \sin^2\theta \ d\varphi^2\right) + Cdt^2 - Ddu^2 \quad (1.6)$$

by transformation T method.

Takeno has used the notation S_0 to denote an arbitrary s.s. space time. In any S_0, a coordinate system, in which the metric is of the form (1.3), a s.s. coordinate system in a wide sense, and the system, in which the metric is of the form (1.4), a s.s. coordinate system in a narrow sense.

Further Takeno denote S_0's whose metric are given by (1.3) or (1.4) in which B is not constant by S_I while an S_0's whose B is a constant by S_{II}. This classification of S_0's into S_I and S_{II}'s has an invariant meaning independent of the special choice of the co-ordinate system.

In this paper use define Canonical co-ordinate system (Ccs), Standard coordinate system for g_{ij} and special coordinate system (Scs) in narrow sense of V_5. Further we proved that, in Scs, when S_0 admitting Ccs and six of the principal invariants are zero, one is non-vanishing (negative) constant, then the S_0 is an S_{II} in V_5.

2. Canonical co-ordinate system

Line element of some S_0 can be brought into the form

$$ds^2 = -Adr^2 - r^2\left(d\theta^2 + \sin^2\theta \ d\varphi^2\right) + Cdt^2 - Ddu^2 \qquad (2.1)$$

where A , C and D are positive functions of r, t and u. In such cases we call the s.s. co-ordinate system in which (2.1) holds a canonical coordinate system.

Theorem: A necessary and sufficient condition that (1.5) can be brought into the form (2.1) by a transformation T is given by

$$B'\dot{B} \neq 0 \quad or \quad B'\hat{B} \neq 0 \qquad (2.2)$$

where a prime, dot and cap mean derivatives with respect to r, t and u respectively.

Proof: Let the line element of an SI be given by (1.5) in which $B \neq$ constant. By non singulartransformation T (1.5) can reduced to EFG co-ordinate system given by,

$$ds^2 = -B\left(d\theta^2 + \sin^2\theta \ d\varphi^2\right) + 2Edrdt + 2Fdrdu + 2Gdtdu \qquad (2.3)$$

From the assumption, B is not constant. Then the condition for T which transforms (2.3) into (2.1) are

$$r = \sqrt{B} \qquad (2.4)$$

$$\frac{\partial \bar{r}}{\partial r}\frac{\partial \bar{t}}{\partial t} + \frac{\partial \bar{r}}{\partial t}\frac{\partial \bar{t}}{\partial r} = 0 \qquad (2.5a)$$

$$\frac{\partial \bar{r}}{\partial r}\frac{\partial \bar{u}}{\partial u}+\frac{\partial \bar{r}}{\partial u}\frac{\partial \bar{u}}{\partial r}=0 \tag{2.5b}$$

$$\frac{\partial u}{\partial u}\frac{\partial \bar{t}}{\partial t}+\frac{\partial \bar{u}}{\partial t}\frac{\partial \bar{t}}{\partial u}=0 \tag{2.5c}$$

i.e.

$$B'\frac{\partial \bar{t}}{\partial t}+\dot{B}\frac{\partial \bar{t}}{\partial r}=0 \tag{2.5d}$$

$$B'\frac{\partial \bar{u}}{\partial u}+\hat{B}\frac{\partial \bar{u}}{\partial r}=0 \tag{2.5e}$$

$$\frac{\partial u}{\partial u}\frac{\partial \bar{t}}{\partial t}+\frac{\partial \bar{u}}{\partial t}\frac{\partial \bar{t}}{\partial u}=0 \tag{2.5f}$$

(2.4) and a non-constant solution $\bar{t}$ and $\bar{u}$ of (2.5) are to define a T satisfying the required conditions.

(i) when (2.2) holds the solution of (2.5) satisfies $\frac{\partial \bar{t}}{\partial t}\neq 0,\ \ \frac{\partial \bar{t}}{\partial r}\neq 0$ and $\frac{\partial \bar{u}}{\partial u}\neq 0,\ \ \frac{\partial \bar{u}}{\partial r}\neq 0$ and from these relation we have $\frac{\partial(\bar{r},\bar{t},\bar{u})}{\partial(r,t,u)}\neq 0$. Therefore the condition (2.2) is sufficient.

(ii) when $B'=0$ and $\dot{B}\neq 0,\ \hat{B}\neq 0$ i.e. when B = B(t , u), we have from (2.4) and (2.5),
$\frac{\partial \bar{r}}{\partial r}=\frac{\partial \bar{t}}{\partial r}=\frac{\partial \bar{u}}{\partial r}=0$ Therefore $\frac{\partial(\bar{r},\bar{t},\bar{u})}{\partial(r,t,u)}=0$ i.e. we cannot obtain non-singular T.

when $B'\neq 0$ and $\dot{B}=0,\ \hat{B}=0$ i.e. when B = B(r,u), we have from (2.4) and (2.5),
$\frac{\partial \bar{t}}{\partial t}=\frac{\partial \bar{u}}{\partial u}=\frac{\partial \bar{r}}{\partial t}=\frac{\partial \bar{r}}{\partial u}=0$ b Therefore $\frac{\partial(\bar{r},\bar{t},\bar{u})}{\partial(r,t,u)}=0$ i.e. again we cannot obtain non-singular T.

3. Standard co-ordinate system for g_{ij} :

Takeno has defined standard coordinate system for g_{ij} as, in a coordinate system, the s.s. line element be given by (1.4) and the following relation hold:

$$\gamma=0 \ \ i.e.\ \ f_5=0 \tag{3.1a}$$

or

$$-2\dot{B}'+\frac{\dot{B}C'}{C}+\frac{B'\dot{A}}{A}+\frac{\dot{B}B'}{B}=0 \tag{3.1b}$$

We define standard coordinate system for g_{ij} in V_5 as follows:

Let, in a coordinate system, the metric of s.s. be given by (1.6) and the following relation hold:

$$\alpha_7=\alpha_9=\alpha_{11}=\alpha_{13}=\alpha_{15}=\alpha_{17}=0 \tag{3.2a}$$

or

$$f_4 = f_5 = f_6 = f_7 = f_8 = f_{11} = 0 \qquad (3.2b)$$

Now, we shall give an example of S_0 having a standard coordinate system for g_{ij} .

we consider a s.s. metric

$$ds^2 = -Adr^2 - B\left(d\theta^2 + \sin^2\theta \; d\varphi^2\right) + Cdt^2 - Ddu^2 \qquad (3.3)$$

Where

$$A = \frac{r}{t-u},\ B = \frac{r^2}{t-u},\ C = \frac{-r^3}{(t-u)^3} + (t-u)e^{\frac{-r}{t-u}},\quad D = \frac{r^3}{(t-u)^3} - (t-u)e^{\frac{-r}{t-u}} + u \qquad (3.4)$$

we observed that f_4 , f_5, f_6, f_7, f_8 and f_{11} are not zero for above metric.

(3.3) transformed by the transformation:

$$r = \bar{r}\,\bar{t}\ , t = \bar{u} + \bar{t}\ , u = \bar{u} \qquad (3.5)$$

Into

$$d\bar{s}^2 = -\bar{t}^2 d\bar{r}^2 - \bar{t}\ \bar{r}^2\left(d\bar{\theta}^2 + \sin^2\bar{\theta}\ \ d\bar{\varphi}^2\right) + \bar{t}\, e^{-\bar{r}} d\bar{t}^2 - \bar{u}d\bar{u}^2 \qquad (3.6)$$

for which we have $\bar{f}_4 = \bar{f}_5 = \bar{f}_6 = \bar{f}_7 = \bar{f}_8 = \bar{f}_{11} = 0$

Thus new coordinate system is standard for g_{ij} .

4. Special co-ordinate system

We define special co-ordinate system (Scs) in V_5 as follows:

Let, in a co-ordinate system, the metric of an s.s. be given by (1.6) and the following relation holds :

$$\alpha_{13} = \alpha_{15} = \alpha_{17} = 0 \qquad (4.1a)$$

or

$$f_6 = f_7 = f_8 = 0 \qquad (4.1b)$$

An example of S_0 having Scs is given below. we consider a s.s. metric in Ccs:

$$ds^2 = -Adr^2 - r^2\left(d\theta^2 + \sin^2\theta \; d\varphi^2\right) + Cdt^2 - Ddu^2 \qquad (4.2)$$

Where

$$A = r(2rt + 3u),\quad C = \frac{-r}{2rt + 3u},\quad D = \frac{1 + u(2rt + 3u)}{2rt + 3u} \qquad (4.3)$$

for which

$\alpha_{13} \neq 0$, $\alpha_{15} \neq 0$, $\alpha_{17} \neq 0$

(4.2) transformed by transformation:

$$r = \bar{r} \;, t = \frac{\bar{t} - \bar{u}}{\bar{r}} + \;, u = \bar{u} \tag{4.4}$$

Into

$$d\bar{s}^2 = -\bar{r}\left(\bar{u} + 2\bar{t}\right)d\bar{r}^2 - \bar{r}^2\left(d\bar{\theta}^2 + \sin^2\bar{\theta}\;\; d\bar{\varphi}^2\right) - \frac{1}{\bar{u} + 2\bar{t}}d\bar{t}^2 - \bar{u}d\bar{u}^2 \tag{4.6}$$

for which we have $\alpha_{13} = \alpha_{15} = \alpha_{17} = 0$.

Thus the new coordinate system is special co-ordinate system.

5. Canonical co-ordinate system for S_{II}

We call a coordinate system in which the line element of an S_{II} is of the form (1.6) in which B = constant a canonical coordinate system of the S_{II} .

In canonical co-ordinate system for S_{II} , we observed that, $f_1 = f_4 = f_5 = f_6 = f_8 = f_{11} = 0$

Thus it is evident that we have :

In Special co-ordinate system any S_{II} any canonical coordinate system is standard for g_{ij} .

6. Principal invariants in Ccs

Theorem: In special coordinate system when S_0 admitting canonical coordinate system and six of the principal invariants are zero, one is non-vanishing constant, then the S_0 is an S_{II} in V_5.

Proof: In Ccs, the non vanishing components of curvature tensor K_{ijkl} and $\alpha_1, \alpha_2, ...\alpha_{19}$ are given by

$$K_{1212} = -\frac{A'r}{2A} = f_1 \;, \quad K_{1414} = \frac{\ddot{A} - C''}{2} - \frac{1}{4}\left[\frac{\dot{A}^2}{A} - \frac{C'^2}{C} - \frac{A'C'}{A} + \frac{\dot{A}\dot{C}}{C} + \frac{\hat{A}\hat{C}}{D}\right] = f_2,$$
$$K_{1515} = \frac{\hat{\hat{A}} - D''}{2} - \frac{1}{4}\left[\frac{\hat{A}^2}{A} - \frac{D'^2}{D} - \frac{A'D'}{A} + \frac{\dot{A}\dot{D}}{D} + \frac{\hat{A}\hat{D}}{D}\right] = f_3, \quad K_{1224} = \frac{r\dot{A}}{2A} = f_4 \;,$$
$$K_{1225} = \frac{r\hat{A}}{2A} = f_5 \;, \quad K_{1415} = \frac{\hat{\dot{A}}}{2} - \frac{1}{4}\left[\frac{\dot{A}\hat{A}}{A} + \frac{\dot{A}\hat{C}}{C} + \frac{\hat{A}\dot{D}}{D}\right] = f_6 \;, K_{1445} = \frac{\hat{C}'}{2} - \frac{1}{4}\left[\frac{\dot{A}C'}{A} + \frac{\hat{C}D'}{D} + \frac{\hat{C}C'}{C}\right] = f_7,$$
$$K_{1545} = \frac{\dot{D}'}{2} - \frac{1}{4}\left[\frac{\dot{D}D'}{D} + \frac{\dot{A}D'}{A} + \frac{C'\dot{D}}{C}\right] = f_8 \;, \quad K_{2424} = \frac{K_{3434}}{\sin^2\theta} = \frac{-rC'}{2A} = f_9 \;, \quad K_{2525} = \frac{K_{3535}}{\sin^2\theta} = \frac{rD'}{2A} = f_{10},$$
$$K_{2425} = \frac{K_{3435}}{\sin^2\theta} = 0 = f_{11} \;, \quad \frac{K_{2323}}{\sin^2\theta} = r^2\left(\frac{1}{A} - 1\right) = f_{12}, \; K_{4545} = \frac{\ddot{D} - \hat{\hat{C}}}{2} + \frac{1}{4}\left[\frac{\hat{C}^2}{C} - \frac{\dot{D}^2}{D} - \frac{\dot{C}\dot{D}}{A} - \frac{C'D'}{C} + \frac{\hat{C}\hat{D}}{D}\right] = f_{13}.$$

$K_{12}^{..12} = K_{13}^{..13} = \alpha_1 = -\frac{A'}{2A^2 r}$, $K_{24}^{..24} = K_{34}^{..34} = \alpha_2 = \frac{-1}{BC} f_9$, $K_{14}^{..14} = \alpha_3 = -\frac{1}{AC} f_2$, $K_{15}^{..15} = \alpha_4 = \frac{1}{AD} f_3$,
$K_{23}^{..23} = \alpha_5 = \frac{1}{B^2} f_{12} = \frac{1}{r^2}\left(\frac{1}{A} - 1\right)$, $K_{25}^{..25} = K_{35}^{..35} = \alpha_6 = \frac{1}{BD} f_{10}$, $K_{12}^{..24} = K_{13}^{..34} = \alpha_7 = \frac{-1}{BC} f_4$,
$K_{24}^{..12} = K_{34}^{..13} = \alpha_8 = \frac{1}{AB} f_4$, $K_{12}^{..25} = K_{13}^{..35} = \alpha_9 = \frac{1}{BD} f_5$, $K_{25}^{..12} = K_{35}^{..13} = \alpha_{10} = \frac{1}{AB} f_5$, $K_{14}^{..15} = \alpha_{13} = \frac{1}{AD} f_6$,
$K_{15}^{..14} = \alpha_{14} = \frac{-1}{AC} f_6$, $K_{15}^{..45} = \alpha_{15} = \frac{-1}{CD} f_8$, $K_{45}^{..15} = \alpha_{16} = \frac{1}{AD} f_8$, $K_{14}^{..45} = \alpha_{17} = \frac{-1}{CD} f_7$,
$K_{45}^{..14} = \alpha_{18} = \frac{-1}{AC} f_7, K_{45}^{..45} = \alpha_{19} = \frac{-1}{CD} f_{13}$.

From above we observed that, the following relations holds

$$\alpha_7 = -\frac{A}{C}\alpha_8, \quad \alpha_9 = \frac{A}{D}\alpha_{10}, \quad \alpha_{13} = -\frac{C}{D}\alpha_{14}, \alpha_{15} = -\frac{A}{C}\alpha_{16} \quad ,\alpha_{17} = -\frac{A}{D}\alpha_{18} .$$

P. O. Bagde and K. T. Thomas [6] obtained the ten principal invariants in the special coordinate system for g_{ij} given below,

$$\{\lambda\} \equiv \lambda^s = (\alpha_3, \alpha_4, \alpha_5, \alpha_{19}, \alpha_1, \alpha_1, \alpha_2, \alpha_2, \alpha_6, \alpha_6)$$

If $\alpha_5 = 0$

we have $\frac{1}{r^2}\left(\frac{1}{A} - 1\right) = 0 \Rightarrow A = 1$

so that $\alpha_1 = \alpha_7 = \alpha_8 = \alpha_9 = \alpha_{10} = \alpha_{13} = \alpha_{14} = 0$ and three of λ^s become 0.

Further if $\alpha_2 = 0$, $\alpha_6 = 0$

$\frac{C'}{2CAr} = 0 \Rightarrow C' = 0 \Rightarrow C$ is constant , $\frac{D'}{2ADr} = 0 \Rightarrow D' = 0 \Rightarrow D$ is constant

so that $\alpha_3 = \alpha_4 = \alpha_{19} = 0$,

which can not the case by virtue of the assumption.

If $\alpha_2 \neq 0$, $\alpha_6 \neq 0$ and $\alpha_3 = 0$, $\alpha_4 = 0$, $\alpha_{19} = 0$.

we have $\alpha_3 = 0 \Rightarrow 2C''C - C'^2 = 0 \Rightarrow C = (ar + b)^2$, $a \neq 0$ and b is functionof t,u.

$\alpha_4 = 0 \Rightarrow 2DD'' - D'^2 = 0 \Rightarrow D = (er + g)^2$, $e \neq 0$ and g is functionof t,u.

$$\therefore ds^2 = -dr^2 - r^2\left(d\theta^2 + \sin^2\theta \; d\varphi^2\right) + (ar + b)^2 dt^2 - (er + g)^2 du^2$$

where $a(\neq 0)$, $e(\neq 0)$ and b, f are function of t.

$$\therefore \{\lambda\} = (0, 0, 0, 0, 0, 0, \alpha_2, \alpha_2, \alpha_6, \alpha_6)$$

where $\alpha_2 = \frac{a}{(ar + b)r} \neq$ constant and $\alpha_6 = \frac{e}{(er + f)r} \neq$ constant

which is again inconsistent with the assumption.

Thus we have $\alpha_5 \neq 0$, and then form the assumption we have following cases:

Case I $\alpha_1 = 0, \alpha_2 = 0, \alpha_6 = 0$
Case II(a) $\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0$
Case II(b) $\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0, \alpha_{19} = 0$
Case II(c) $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_{19} = 0$
Case III(a) $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 = 0, \alpha_6 = 0$
Case III(b) $\alpha_1 = 0, \alpha_3 = 0, \alpha_6 = 0, \alpha_{19} = 0$
Case III(c) $\alpha_1 = 0, \alpha_4 = 0, \alpha_6 = 0, \alpha_{19} = 0$
Case IV(a) $\alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0, \alpha_6 = 0$
Case IV(b) $\alpha_2 = 0, \alpha_3 = 0, \alpha_6 = 0, \alpha_{19} = 0$
Case IV(c) $\alpha_2 = 0, \alpha_4 = 0, \alpha_6 = 0, \alpha_{19} = 0$

If $\alpha_5 = \alpha_5(r)$, then A also become function of r.

Case I $\alpha_1 = 0, \alpha_2 = 0, \alpha_6 = 0$

$\Rightarrow A' = 0,\ C' = 0,\ D' = 0$

Accordingly, $\alpha_3 = 0, \alpha_4 = 0, \alpha_{19} = 0$

Thus we have,

$$ds^2 = -Pdr^2 - r^2\left(d\theta^2 + \sin^2\theta\ d\varphi^2\right) + C(t,u)dt^2 - D(t,u)du^2 \quad , \ (P = \text{constant} \neq 1)$$

and $\{\lambda\} = (0, 0, 0, 0, 0, 0, 0, 0, 0, \alpha_5)$ where $\alpha_5 = \frac{1}{r^2}\left(\frac{1}{P} - 1\right) \neq$ constant.

This is contradiction to the assumption.

Case II(a) $\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0$.

$\Rightarrow A' = 0,\ C' = 0$

Now $\alpha_4 = 0 \Rightarrow 2DD'' - D'^2 = 0 \Rightarrow D = (er + g)^2$, $e \neq 0$ and g is function of t,u.

$$\alpha_6 = \frac{e}{P(er+g)r} \neq \text{constant} \ \text{and} \ \alpha_{19} = \frac{-1}{CD}\left(\frac{\ddot{D}}{2} - \frac{\dot{D}^2}{4D}\right) \neq \text{constant}.$$

$$\therefore ds^2 = -Pdr^2 - r^2\left(d\theta^2 + \sin^2\theta\ d\varphi^2\right) + C(t,u)dt^2 - (er+g)^2 du^2 \quad , \ (P = \text{constant} \neq 1)$$

and $\{\lambda\} = (0, 0, 0, 0, 0, 0, \alpha_6, \alpha_6, \alpha_{19}, \alpha_5)$ where $\alpha_5 = \frac{1}{r^2}\left(\frac{1}{P} - 1\right) \neq$ constant,

$\alpha_6 \neq$ constant and $\alpha_{19} \neq$ constant

This is contradiction to the assumption.

Case II(b) $\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0, \alpha_{19} = 0$

$\Rightarrow A' = 0,\ C' = 0$

$\alpha_{19} = 0 \Rightarrow 2D\ddot{D} - \dot{D}^2 = 0 \Rightarrow D = (kt + l)^2,\ k \neq 0$ and l is function of r, u.

Now $\alpha_4 = \frac{1}{PD}\left(\frac{D''}{2} - \frac{D'^2}{4D}\right) \neq$ constant i.e. α_4 is a function of (r, t, u) and $\alpha_6 = \frac{D'}{2rPD} \neq$ constant.

$\therefore ds^2 = -Pdr^2 - r^2(d\theta^2 + \sin^2\theta\ d\varphi^2) + C(t,u)dt^2 - (kt + l)^2 du^2 \quad , (P = \text{constant} \neq 1)$

and $\{\lambda\} = (0, 0, 0, 0, 0, 0, \alpha_4, \alpha_6, \alpha_6, \alpha_5)$ where $\alpha_5 \neq$ constant , $\alpha_4 \neq$ constant and $\alpha_6 \neq$ constant

This is contradiction to the assumption.

Case II(c) $\alpha_1 = 0, \alpha_2 = 0, \alpha_4 = 0, \alpha_{19} = 0$

$\Rightarrow A' = 0,\ C' = 0 \quad \Rightarrow \alpha_3 = 0$

Now $\alpha_4 = 0 \Rightarrow 2DD'' - D'^2 = 0 \Rightarrow D = (er + g)^2,\ e \neq 0$ and g is function of t, u.

$\therefore \alpha_6 = \frac{D'}{2rPD} \neq \text{constant}.$

$\therefore ds^2 = -Pdr^2 - r^2(d\theta^2 + \sin^2\theta\ d\varphi^2) + C(t,u)dt^2 - (er + g)^2 du^2 \quad , (P = \text{constant} \neq 1)$

and $\{\lambda\} = (0, 0, 0, 0, 0, 0, 0, \alpha_6, \alpha_6, \alpha_5)$ where $\alpha_5 \neq$ constant and $\alpha_6 \neq$ constant

This is contradiction to the assumption.

Case III(a) $\alpha_1 = 0, \alpha_3 = 0, \alpha_4 = 0, \alpha_6 = 0$

$\Rightarrow A' = 0,\ D' = 0$

Now $\alpha_3 = 0 \Rightarrow 2C''C - C'^2 = 0 \Rightarrow C = (ar + b)^2,\ a \neq 0$ and b is function of t, u.

$\alpha_2 = \frac{C'}{2rCP} \neq$ constant and $\alpha_{19} = \frac{-1}{CD} f_{13} \neq$ constant

$\therefore ds^2 = -Pdr^2 - r^2(d\theta^2 + \sin^2\theta\ d\varphi^2) + (ar + b)^2 dt^2 - D(t,u)du^2 \quad , (P = \text{constant} \neq 1)$

and $\{\lambda\} = (0, 0, 0, 0, 0, 0, \alpha_2, \alpha_2, \alpha_{19}, \alpha_5)$ where $\alpha_5 \neq$ constant , $\alpha_2 \neq$ constant and $\alpha_{19} \neq$ constant

This is contradiction to the assumption.

Case III(b) $\alpha_1 = 0, \alpha_3 = 0, \alpha_6 = 0, \alpha_{19} = 0$

$\Rightarrow A' = 0,\ D' = 0 \Rightarrow \alpha_4 = 0$

Now $\alpha_3 = 0 \Rightarrow 2C''C - C'^2 = 0 \Rightarrow C = (ar + b)^2,\ a \neq 0$ and b is function of t, u.

$\alpha_2 = \dfrac{C'}{2rCP} \neq \text{constant}$

$\therefore ds^2 = -Pdr^2 - r^2\left(d\theta^2 + \sin^2\theta \ d\varphi^2\right) + (ar+b)^2 dt^2 - D(t,u)du^2 \quad , \ (P = \text{constant} \neq 1)$

and $\{\lambda\} = (0, 0, 0, 0, 0, 0, 0, \alpha_2, \alpha_2, \alpha_5)$ where $\alpha_5 \neq$ constant and $\alpha_2 \neq$ constant .

This is contradiction to the assumption.

Case III(c) $\alpha_1 = 0, \alpha_4 = 0, \alpha_6 = 0, \alpha_{19} = 0$

$\Rightarrow A' = 0, \ D' = 0$

Now $\alpha_{19} = 0 \Rightarrow 2C\hat{\hat{C}} - \hat{C}^2 = 0 \Rightarrow C = (mu+n)^2, \ m \neq 0$ and n is function of r,t.

$\alpha_2 = \dfrac{C'}{2rCP} \neq \text{constant}$ and $\alpha_3 = \dfrac{-1}{PC} f_2 \neq \text{constant}$

$\therefore \{\lambda\} = (0, 0, 0, 0, 0, 0, \alpha_2, \alpha_2, \alpha_3, \alpha_5)$ where $\alpha_5 \neq$ constant, $\alpha_2 \neq$ constant and $\alpha_3 \neq$ constant.

This is contradiction to the assumption.

Case IV (a) $\alpha_2 = 0, \alpha_3 = 0, \ \alpha_4 = 0, \alpha_6 = 0$

$\Rightarrow C' = 0 \, , D' = 0 \Rightarrow \alpha_{19} = 0$

Now $\alpha_3 = 0 \Rightarrow 2A\ddot{A} - \dot{A}^2 = 0 \Rightarrow A = (qt+s)^2, q \neq 0$ and s is a function of r, u.

$\therefore \alpha_1 \neq$ constant.

$\{\lambda\} = (0, 0, 0, 0, 0, 0, 0, \alpha_1, \alpha_1, \alpha_5)$ where $\alpha_1 \neq$ constant and $\alpha_5 \neq$ constant.

This is a contradiction.

Case IV (b) $\alpha_2 = 0, \alpha_3 = 0, \ \alpha_6 = 0, \alpha_{19} = 0$

$\Rightarrow C' = 0 \, , D' = 0$

Now $\alpha_3 = 0 \Rightarrow 2A\ddot{A} - \dot{A}^2 = 0 \Rightarrow A = (qt+s)^2, q \neq 0$ and s is a function of r, u.

$\therefore \alpha_1 \neq$ constant , $\alpha_4 \neq$ constant

$\{\lambda\} = (0, 0, 0, 0, 0, 0, \alpha_1, \alpha_1, \alpha_4, \alpha_5)$ where $\alpha_1 \neq$ constant, $\alpha_4 \neq$ constant and $\alpha_5 \neq$ constant.

This is a contradiction.

Case IV (c) $\alpha_2 = 0, \alpha_4 = 0, \ \alpha_6 = 0, \alpha_{19} = 0$

$\Rightarrow C' = 0 \, , D' = 0$

Now $\alpha_4 = 0 \Rightarrow 2A\hat{\hat{A}} - \hat{A}^2 = 0 \Rightarrow A = (vu + w)^2, v \neq 0$ and w is a function of r, t.

$\therefore \alpha_1 \neq$ constant, $\alpha_3 \neq$ constant

$\{\lambda\} = (0, 0, 0, 0, 0, 0, \alpha_1, \alpha_1, \alpha_3, \alpha_5)$ where $\alpha_1 \neq$ constant, $\alpha_3 \neq$ constant and $\alpha_5 \neq$ constant.

This is a contradiction.

Thus, if $\alpha_5 = \alpha_5(r)$, we get contradiction.

Consider $\dot{\alpha}_5 \neq 0, \hat{\alpha}_5 = 0$

Then from the assumption we have following cases:

Case I $\alpha_3 =$ constant $\neq 0$, $\alpha_4 =$ constant $\neq 0$, $\alpha_{19} =$ constant $\neq 0$ and $\alpha_1 = \alpha_2 = \alpha_6 = 0$.

Case II(a) $\alpha_1 =$ constant $\neq 0$, $\alpha_3 =$ constant $\neq 0$ and $\alpha_2 = \alpha_4 = \alpha_6 = \alpha_{19} = 0$.

Case II(b) $\alpha_1 =$ constant $\neq 0$, $\alpha_4 =$ constant $\neq 0$ and $\alpha_2 = \alpha_3 = \alpha_6 = \alpha_{19} = 0$.

Case II(c) $\alpha_1 =$ constant $\neq 0$, $\alpha_{19} =$ constant $\neq 0$ and $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_6 = 0$.

Case III(a) $\alpha_2 =$ constant $\neq 0$, $\alpha_3 =$ constant $\neq 0$ and $\alpha_1 = \alpha_4 = \alpha_6 = \alpha_{19} = 0$.

Case III(b) $\alpha_2 =$ constant $\neq 0$, $\alpha_4 =$ constant $\neq 0$ and $\alpha_1 = \alpha_3 = \alpha_6 = \alpha_{19} = 0$.

Case III(c) $\alpha_2 =$ constant $\neq 0$, $\alpha_{19} =$ constant $\neq 0$ and $\alpha_1 = \alpha_3 = \alpha_4 = \alpha_6 = 0$.

Case IV(a) $\alpha_3 =$ constant $\neq 0$, $\alpha_6 =$ constant $\neq 0$ and $\alpha_1 = \alpha_2 = \alpha_4 = \alpha_{19} = 0$.

Case IV(b) $\alpha_4 =$ constant $\neq 0$, $\alpha_6 =$ constant $\neq 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_{19} = 0$.

Case IV(c) $\alpha_6 =$ constant $\neq 0$, $\alpha_{19} =$ constant $\neq 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$.

Now

Case I $\alpha_3 =$ constant $\neq 0$, $\alpha_4 =$ constant $\neq 0$, $\alpha_{19} =$ constant $\neq 0$ and $\alpha_1 = \alpha_2 = \alpha_6 = 0$.

$\alpha_1 = 0 \Rightarrow A' = 0$, $\alpha_2 = 0 \Rightarrow C' = 0$ and $\hat{\alpha}_5 = 0 \Rightarrow \hat{A} = 0$

$$\Rightarrow \alpha_3 = \frac{1}{AC}\left(-\frac{\ddot{A}}{2} + \frac{1}{4}\left[\frac{\dot{A}^2}{A} + \frac{\dot{A}\dot{C}}{C}\right]\right) \neq \text{constant.}$$

This is a contradiction.

Case II(a) $\alpha_1 =$ constant $\neq 0$, $\alpha_3 =$ constant $\neq 0$ and $\alpha_2 = \alpha_4 = \alpha_6 = \alpha_{19} = 0$.

$\alpha_2 = 0 \Rightarrow C' = 0$, $\alpha_6 = 0 \Rightarrow D' = 0$ and $\hat{\alpha}_5 = 0 \Rightarrow \hat{A} = 0$

$$\Rightarrow \alpha_3 = \frac{1}{AC}\left(-\frac{\ddot{A}}{2} + \frac{1}{4}\left[\frac{\dot{A}^2}{A} + \frac{\dot{A}\dot{C}}{C}\right]\right) \neq \text{constant.}$$

This is a contradiction.

Case II(b) $\alpha_1 =$ constant $\neq 0$, $\alpha_4 =$ constant $\neq 0$ and $\alpha_2 = \alpha_3 = \alpha_6 = \alpha_{19} = 0$.

$\alpha_2 = 0 \Rightarrow C' = 0$, $\alpha_6 = 0 \Rightarrow D' = 0$ and $\hat{\alpha}_5 = 0 \Rightarrow \hat{A} = 0$

$\Rightarrow \alpha_4 = -\frac{\dot{A}\dot{D}}{4ADC} \neq$ constant.

This is a contradiction.

Case II(c) $\alpha_1 =$ constant $\neq 0$, $\alpha_{19} =$ constant $\neq 0$ and $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_6 = 0$.

$\alpha_2 = 0 \Rightarrow C' = 0$, $\alpha_6 = 0 \Rightarrow D' = 0$ and $\dot{\alpha}_5 \neq 0 \Rightarrow \dot{A} \neq 0$

$\Rightarrow \alpha_4 = 0 \Rightarrow -\frac{\dot{A}\dot{D}}{4ADC} = 0 \Rightarrow \dot{D} = 0$.

$\therefore \alpha_{19} = -\frac{1}{CD}\left(-\frac{\hat{\hat{C}}}{2} + \frac{1}{4}\left[\frac{\hat{C}^2}{C} + \frac{\hat{C}\hat{D}}{D}\right]\right) \neq$ constant.

This is a contradiction.

Case III(a) $\alpha_2 =$ constant $\neq 0$, $\alpha_3 =$ constant $\neq 0$ and $\alpha_1 = \alpha_4 = \alpha_6 = \alpha_{19} = 0$.

$\alpha_1 = 0 \Rightarrow A' = 0$, $\hat{\alpha}_5 = 0 \Rightarrow \hat{A} = 0$ and $\dot{\alpha}_5 \neq 0 \Rightarrow \dot{A} \neq 0$ i.e.A is function of r.

$\Rightarrow \alpha_3 = \frac{1}{AC}\left(\frac{C'' - \ddot{A}}{2} + \frac{1}{4}\left[\frac{\dot{A}^2}{A} - \frac{C'^2}{C} + \frac{\dot{A}\dot{C}}{C}\right]\right) \neq$ constant.

This is a contradiction.

Case III(b) $\alpha_2 =$ constant $\neq 0$, $\alpha_4 =$ constant $\neq 0$ and $\alpha_1 = \alpha_3 = \alpha_6 = \alpha_{19} = 0$.

$\alpha_1 = 0 \Rightarrow A' = 0$, $\hat{\alpha}_5 = 0 \Rightarrow \hat{A} = 0$ and $\dot{\alpha}_5 \neq 0 \Rightarrow \dot{A} \neq 0$ i.e.A is function of r.

$\alpha_2 = \frac{C'}{2rCA} \neq$ constant.

This is a contradiction.

Case III(c) $\alpha_2 =$ constant $\neq 0$, $\alpha_{19} =$ constant $\neq 0$ and $\alpha_1 = \alpha_3 = \alpha_4 = \alpha_6 = 0$.

$\alpha_1 = 0 \Rightarrow A' = 0$, $\hat{\alpha}_5 = 0 \Rightarrow \hat{A} = 0$ and $\dot{\alpha}_5 \neq 0 \Rightarrow \dot{A} \neq 0$ i.e.A is function of r.

$\alpha_2 = \frac{C'}{2rCA} \neq$ constant.

This is a contradiction.

Case IV(a) $\alpha_3 =$ constant $\neq 0$, $\alpha_6 =$ constant $\neq 0$ and $\alpha_1 = \alpha_2 = \alpha_4 = \alpha_{19} = 0$.

$\alpha_1 = 0 \Rightarrow A' = 0$, $\alpha_2 = 0 \Rightarrow C' = 0$, $\hat{\alpha}_5 = 0 \Rightarrow \hat{A} = 0$ and $\dot{\alpha}_5 \neq 0 \Rightarrow \dot{A} \neq 0$ i.e. A is function of r.

$$\Rightarrow \alpha_3 = \frac{1}{AC}\left(-\frac{\ddot{A}}{2} + \frac{1}{4}\left[\frac{\dot{A}^2}{A} + \frac{\dot{A}\dot{C}}{C}\right]\right) \neq \text{constant.}$$

This is a contradiction.

Case IV(b) $\alpha_4 =$ constant $\neq 0$, $\alpha_6 =$ constant $\neq 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_{19} = 0$.

$\alpha_1 = 0 \Rightarrow A' = 0$, $\alpha_2 = 0 \Rightarrow C' = 0$, $\hat{\alpha}_5 = 0 \Rightarrow \hat{A} = 0$ and $\dot{\alpha}_5 \neq 0 \Rightarrow \dot{A} \neq 0$ i.e. A is function of r.

$$\Rightarrow \alpha_4 = \frac{1}{AD}\left(\frac{\ddot{D}}{2} - \frac{1}{4}\left[\frac{D'^2}{D} + \frac{\dot{A}\dot{D}}{C}\right]\right) \neq \text{constant.}$$

This is a contradiction.

Case IV(c) $\alpha_6 =$ constant $\neq 0$, $\alpha_{19} =$ constant $\neq 0$ and $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$.

$\alpha_1 = 0 \Rightarrow A' = 0$, $\hat{\alpha}_5 = 0 \Rightarrow \hat{A} = 0$ and $\dot{\alpha}_5 \neq 0 \Rightarrow \dot{A} \neq 0$ i.e.A is function of r.

$$\alpha_6 = \frac{D'}{2rAD} \neq \text{constant.}$$

This is a contradiction.

Thus we get a contradiction when $\dot{\alpha}_5 \neq 0$, $\hat{\alpha}_5 = 0$.

In the similar fashion we can prove the contradiction when $\dot{\alpha}_5 = 0$, $\hat{\alpha}_5 \neq 0$.

Hence, S_0 cannot be S_I.

$\therefore$ If six of the $\lambda^{,s}$ are zero and one is a non-vanishing (negative) constant, then, S_0 is an S_{II}.

Conclusion

In special coordinate system any S_{II} any canonical coordinate system is standard for g_{ij}. In special coordinate system when S_0 admitting canonical coordinate system and six of the principal invariants are zero, one is non-vanishing constant, then the S_0 is S_{II} in V_5.

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