Article

Scale Independent Unified Quark Physics

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Abstract

Based on available particle data, up quark mass is 2.15 MeV/c^2, down quark mass is 4.7 MeV/c^2 and their mass ratio is 0.46(5). In an earlier paper, the authors proposed a simple method for estimating the six quark masses. In this paper, the authors propose very simple methods for estimating the nuclear stability and nuclear binding energy and successfully extend the quark model to low energy scale nuclear physics. Topics covered are: 1) Hint of final unification and Up and Down quark rest masses, 2) Strong coupling constant, 3) Nucleon rest masses, 4) Nuclear stability, 5) Nuclear binding energy, 6) Strong interaction range, 7) Nuclear charge radius, 8) RMS charge radius of proton, 9) RMS charge radii of atomic nuclei and 10) Magic numbers. At the stable mass number of Z, nuclear binding energy seems to be proportional to the sum of $2Z$ number of up quarks rest energy and Z number of down quarks rest energy. It is interesting to note that binding energy near to stable mass number seems to be independent of the mass number. Finally an attempt is made to fit the SEMF energy coefficients.

Keywords: Strong interaction, gravitational constant, up quark, down quark, strong coupling constant, nuclear stability, nuclear binding energy, nuclear charge radii, magic numbers.

1. Introduction

Denis Lacroix states in [1] that – "two fundamental questions of present days nuclear physics are: (i) How to understand the very rich structure of atomic nuclei in terms of interaction between nucleons? (ii) How to relate the strong nuclear interaction to the underlying fundamental Chromodynamics (QCD) that governs the physics of quarks and gluons? These two questions illustrate the many facets of nucleon-nucleon interactions. "Low energy" nuclear scientists mainly address (i) and often consider the strong interaction as a "fundamental" interaction and nucleons as elementary (often pointlike) particles. From the "High energy" nuclear physics point of view, nucleons, being formed by quarks and gluons, can obviously not be considered as elementary particles and the strong interaction itself should be understood from the more fundamental Standard Model. The nuclear force is at the crossroad of these two visions. Recently, important progresses have been made in understanding the nuclear strong interaction directly from QCD. Conjointly, new experimental results have pointed out our lack of knowledge of the interaction in the nuclear medium". Finally he concludes that, "Recent achievements in nuclear forces theory open new perspectives for the next decade of low energy nuclear physics, bringing together people from very different communities. Although many

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developments remain to be done, the possibility to directly use QCD to describe nuclear system is a major challenge that is within reach".

Based on particle data group [2], up quark mass is $m_u \cong 2.15 \text{ MeV}/c^2$ and down quark mass is $m_d \cong 4.7 \text{ MeV}/c^2$.Up and down quark mass ratio is 0.46(5).In the earlier published paper [3] titled: "Super symmetry in strong and weak interactions", the authors proposed a simple method for estimating the six quark masses. With reference to the earlier proposed results, up quark mass is $m_u \cong 4.4 \text{ MeV}/c^2$ and down quark mass is $m_d \cong 9.473 \text{ MeV}/c^2$. By considering the up and down quark rest masses, in this paper, the authors proposed very simple methods for estimating the nuclear stability and nuclear binding energy [4] and successfully extended the quark model to low energy scale nuclear physics.Topics covered are: 1) Hint of final unification and Up and Down quark rest masses, 2) Strong coupling constant, 3) Nucleon rest masses, 4) Nuclear stability, 5) Nuclear binding energy, 6) Strong interaction range, 7) Nuclear charge radius, 8) RMS charge radius of proton, 9) RMS charge radii of atomic nuclei and 10) Magic numbers.

2. Role of Down and Up quark masses in understanding nuclear stability

Despite the mutual electromagnetic repulsion of protons, a stronger attractive force was postulated to explain how the atomic nucleus was bound together. This hypothesized force was called the 'strong force', which was believed to be a fundamental force that acted on nucleons: the protons and neutrons that make up the nucleus. It was later discovered that protons and neutrons were not fundamental particles, but were made up of constituent particles called 'quarks' [5]. In particle physics, the 'strong interaction' is the mechanism responsible for the strong nuclear force. It is approximately 100 times stronger than electromagnetism, a million times stronger than the weak force interaction. It ensures the stability of ordinary nuclear matter that constitutes observable neutrons and protons. It is also believed that, 'residual strong force' plays a key role in the context of binding protons and neutrons together to form atoms. Clearly speaking, it is the residuum of the strong interaction between the up and down quarks that make up the protons and neutrons.

In the semi empirical mass formula [4], by maximizing $B(A,Z)$ with respect to Z, we find the number of protons Z of the stable nucleus of atomic weight A as,

$$Z \approx \frac{A}{1+\left(a_c/2a_a\right)A^{2/3}}. \tag{1}$$

This is roughly $A/2$ for light nuclei, but for heavy nuclei there is an even better agreement with nature. By substituting the above value of Z back into B one obtains the binding energy as a function of the atomic weight, $B(A)$. Maximizing $B(A)/A$ with respect to A gives the nucleus which is most strongly bound and most stable. Considering the up and down quark masses, and without considering the semi empirical mass formula it is also possible to show that,

$$A_{stable} \cong 2Z + \frac{(Z\alpha_s)^2}{2} \tag{2}$$

where, α_s is the strong coupling constant [5] and A_{stable} represents the estimated stable mass numbers of Z. It is possible to guess that,

$$\frac{1}{\alpha_s} \cong \exp\left(\frac{m_d}{m_u}\right) \cong 8.90 \to \alpha_s \cong 0.11236 \tag{3}$$

where $m_u \cong 2.15 \text{ MeV}/c^2$ and $m_d \cong 4.7 \text{ MeV}/c^2$. This value of the strong coupling constant can be compared with the current estimates of $\alpha_s \cong 0.1186$. For the proposed nuclear stability data see columns 1 and 2 of table-1.

3. Role of Down & Up quark masses in understanding nuclear binding energy

In this section the authors proposed a very simple method for understanding the nuclear binding energy with one energy constant. One important point to be noted here is that, as per the quark model, proton constitutes 3 quarks as (U,U,D). For Z numbers of participating protons, number of participating up quarks can be $2Z$ and number of participating down quarks can be Z. At the stable mass number, it is possible to show that,

$$(B)_{Stable} \cong kZ\left[2m_uc^2 + m_dc^2\right] \tag{4}$$

where k is a coefficient that seems to be related with proton number and the down and up quark mass ratio. For the observed data it can be suggested that,

$$\left.\begin{array}{l} \underline{\text{Case-1}}: Z \cong 2 \text{ to } 30,\ \text{k} \cong \left(\frac{Z}{30}\right)^{\frac{1}{6}}\left(\frac{m_d}{m_u}\right) \\ \underline{\text{Case-2}}: Z >= 30,\ \text{k} \cong \left(\frac{m_d}{m_u}\right) \cong 2.186 \end{array}\right\} \tag{5}$$

Here interesting point to be noted is that, binding energy near to stable mass number is practically independent of mass number! See the following table-1 for nuclear binding energy close to the stable mass number of Z. Above and below the stable mass number, to a very good approximation, nuclear binding energy can be expressed in the following way.

$$\left.\begin{array}{l} \text{If } (\text{A>A}_{Stable}),\ (B)_A \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{2}{3}}(B)_{Stable} \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{2}{3}} kZ\left[2m_uc^2 + m_dc^2\right] \\ \text{If } (\text{A<A}_{Stable}),\ (B)_A \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{4}{3}}(B)_{Stable} \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{4}{3}} kZ\left[2m_uc^2 + m_dc^2\right] \end{array}\right\} \tag{6}$$

Obtained nuclear binding can be compared with results of the standard form of the following semi empirical formula:

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \tag{7}$$

$$\text{where } \begin{cases} a_v \cong 15.78 \text{ MeV}, a_s \cong 18.34 \text{ MeV}, a_c \cong 0.71 \text{ MeV}, \\ a_a \cong 23.21 \text{ MeV}, a_p \cong 12.0 \text{ MeV}. \end{cases}$$

In the following table-1, column-1 represents proton number, column-2 represents estimated stable mass number with the proposed relations (2,3), column-3 represents the estimated value of k from relation (5), column-4 represents the neutron number, column-5 represents the mass number, column-6 represents binding energy calculated from SEMF relation (7) with current energy coefficients and column-7 represents binding energy calculated with proposed relations (4,5).

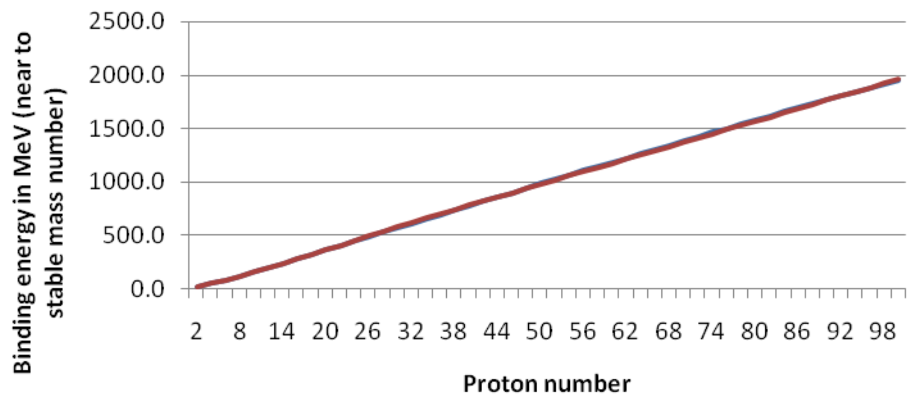
Table 1. To fit the nuclear binding energy near the estimated mean stable mass number of Z

Proton number	Estimated stable mass number	Estimated value of k	Neutron number	Mass number	Binding energy in MeV form SEMF Relation(7)	Proposed binding energy in MeV Relation(4)
2	4	1.3920	2	4	22.0	25.1
4	8	1.5625	4	8	52.9	56.2
6	12	1.6717	6	12	87.4	90.3
8	16	1.7538	8	16	123.2	126.3
10	21	1.8203	11	21	167.5	163.8
12	25	1.8764	13	25	204.7	202.7
14	29	1.9252	15	29	241.6	242.6
16	34	1.9686	18	34	290.8	283.5
18	38	2.0076	20	38	327.2	325.2
20	43	2.0432	23	43	371.6	367.8
22	47	2.0759	25	47	407.5	411.0
24	52	2.1062	28	52	454.6	454.9
26	56	2.1345	30	56	489.6	499.5
28	61	2.1610	33	61	532.5	544.6
30	66	2.1864	36	66	577.9	590.2
32	70	2.1864	38	70	611.7	629.6
34	75	2.1864	41	75	653.3	668.9
36	80	2.1864	44	80	697.0	708.3
38	85	2.1865	47	85	737.6	747.6
40	90	2.1865	50	90	780.2	787.0
42	95	2.1865	53	95	819.8	826.3
44	100	2.1866	56	100	861.2	865.7

46	105	2.1866	59	105	899.8	905.0
48	111	2.1866	63	111	947.7	944.4
50	116	2.1867	66	116	987.5	983.7
52	121	2.1867	69	121	1024.6	1023.1
54	126	2.1867	72	126	1063.4	1062.4
56	132	2.1868	76	132	1108.7	1101.7
58	137	2.1868	79	137	1144.4	1141.1
60	143	2.1868	83	143	1188.5	1180.4
62	148	2.1869	86	148	1225.3	1219.8
64	154	2.1869	90	154	1268.2	1259.1
66	159	2.1869	93	159	1302.1	1298.5
68	165	2.1870	97	165	1343.9	1337.8
70	171	2.1861	101	171	1385.1	1377.2
72	177	2.1861	105	177	1425.7	1416.5
74	183	2.1861	109	183	1465.7	1455.9
76	188	2.1861	112	188	1499.2	1495.2
78	194	2.1861	116	194	1538.2	1534.6
80	200	2.1861	120	200	1576.7	1573.9
82	206	2.1861	124	206	1614.6	1613.3
84	213	2.1861	129	213	1657.5	1652.6
86	219	2.1861	133	219	1694.3	1692.0
88	225	2.1861	137	225	1730.7	1731.3
90	231	2.1861	141	231	1766.5	1770.7
92	237	2.1861	145	237	1801.9	1810.0
94	244	2.1862	150	244	1843.4	1849.4
96	250	2.1862	154	250	1877.8	1888.7
98	257	2.1862	159	257	1916.5	1928.1
100	263	2.1862	163	263	1949.9	1967.4

See the following figure-1 for nuclear binding energy near to estimated stable mass number. In the figure-1, red line represents the estimated binding energy and blue line represents the nuclear binding energy calculated from the semi empirical mass formula with the current recommended energy coefficients.

Estimated stable mass number of $Z = 50$ is $A_{Stable} = 116$.With this stable mass number- an attempt is made to fit the nuclear binding energy with relations (5 and 6). See the following table-2 and related figure-2. In the figure, red curve represents the estimated binding energy from relation (6) and blue curve represents the nuclear binding energy calculated from the semi empirical mass formula with the current recommended energy coefficients (7).

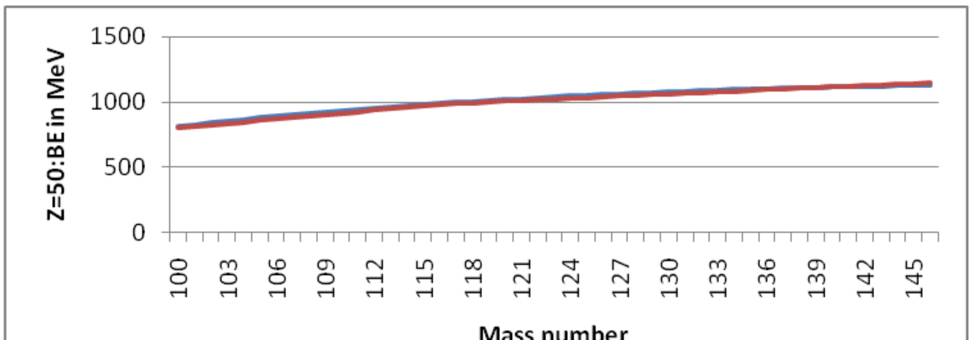


(Fig-1)

Table 2. To fit the SEMF binding energy of $Z = 50$

Proton number	Stable mass number	Assumed value of k	Neutron number	Mass number	Binding energy in MeV from SEMF Relation (7)	Binding energy in MeV calculated with relation (6)
50	116	2.187	50	100	809.3	807.1
50	116	2.187	51	101	822.3	817.9
50	116	2.187	52	102	837.2	828.7
50	116	2.187	53	103	849.2	839.5
50	116	2.187	54	104	863.2	850.4
50	116	2.187	55	105	874.5	861.3
50	116	2.187	56	106	887.6	872.3
50	116	2.187	57	107	898.1	883.3
50	116	2.187	58	108	910.4	894.3
50	116	2.187	59	109	920.1	905.4
50	116	2.187	60	110	931.8	916.5
50	116	2.187	61	111	940.7	927.6
50	116	2.187	62	112	951.6	938.7
50	116	2.187	63	113	960.0	949.9
50	116	2.187	64	114	970.2	961.2
50	116	2.187	65	115	977.9	972.4
50	116	2.187	66	116	987.5	983.7
50	116	2.187	67	117	994.6	989.3
50	116	2.187	68	118	1003.6	995.0
50	116	2.187	69	119	1010.1	1000.6
50	116	2.187	70	120	1018.5	1006.2

50	116	2.187	71	121	1024.4	1011.8
50	116	2.187	72	122	1032.3	1017.3
50	116	2.187	73	123	1037.8	1022.9
50	116	2.187	74	124	1045.1	1028.4
50	116	2.187	75	125	1050.1	1033.9
50	116	2.187	76	126	1056.9	1039.5
50	116	2.187	77	127	1061.4	1044.9
50	116	2.187	78	128	1067.8	1050.4
50	116	2.187	79	129	1071.8	1055.9
50	116	2.187	80	130	1077.7	1061.3
50	116	2.187	81	131	1081.4	1066.8
50	116	2.187	82	132	1086.9	1072.2
50	116	2.187	83	133	1090.1	1077.6
50	116	2.187	84	134	1095.2	1083.0
50	116	2.187	85	135	1098.0	1088.4
50	116	2.187	86	136	1102.7	1093.8
50	116	2.187	87	137	1105.1	1099.1
50	116	2.187	88	138	1109.4	1104.4
50	116	2.187	89	139	1111.5	1109.8
50	116	2.187	90	140	1115.5	1115.1
50	116	2.187	91	141	1117.2	1120.4
50	116	2.187	92	142	1120.8	1125.7
50	116	2.187	93	143	1122.3	1131.0
50	116	2.187	94	144	1125.6	1136.2



(Fig-2)

4. Discussion

With reference to quark soup, understanding nuclear binding energy is very critical and very interesting at fundamental level. In this critical moment, proposed relations connected with strong interaction and nuclear binding energy scheme can be recommended for further research. If one is willing to consider the up quark mass estimate as $m_u \approx 4.4 \text{ MeV}/c^2$ and down quark mass estimate as $m_d \approx 9.473 \text{ MeV}/c^2$ it is possible to show that, ***starting from*** $Z = 30$***, near to the stable***

mass number, proportionality coefficient being "$(m_d/2m_u)$", nuclear binding energy is equal to the sum of rest energies of $(2Zm_uc^2)$ and (Zm_dc^2). See the following relations for nuclear binding energy with suggested up and down quark masses.

$$(B)_{Stable} \cong kZ\left[2m_uc^2 + m_dc^2\right] \tag{8}$$

where k is a coefficient that seems to be related with proton number and the ratio of 'one down quark mass'and 'two up quark masses'. Here $m_u \cong 4.4 \text{ MeV}/c^2$ and $m_d \cong 9.473 \text{ MeV}/c^2$. For the observed data, now it can be suggested that,

$$\left.\begin{array}{l} \underline{\text{Case-1}}: Z \cong 2 \text{ to } 30,\ \text{k} \cong \left(\frac{Z}{30}\right)^{\frac{1}{6}}\left(\frac{m_d}{2m_u}\right) \\ \underline{\text{Case-2}}: Z >= 30,\ \text{k} \cong \left(\frac{m_d}{2m_u}\right) \cong 1.0765 \end{array}\right\} \tag{9}$$

Above and below the stable mass number, to a very good approximation, nuclear binding energy can be expressed in the following way.

$$\left.\begin{array}{l} \text{If } \left(\text{A}>=\text{A}_{Stable}\right),\ (B)_A \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{2}{3}}(B)_{Stable} \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{2}{3}} kZ\left[2m_uc^2 + m_dc^2\right] \\ \text{If } \left(\text{A}<\text{A}_{Stable}\right),\ (B)_A \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{4}{3}}(B)_{Stable} \cong \left(\frac{A}{A_{Stable}}\right)^{\frac{4}{3}} kZ\left[2m_uc^2 + m_dc^2\right] \\ \text{where } A_{stable} \cong 2Z + \left[\left(\frac{m_u}{m_d}\right)(Z\alpha_s)^2\right] \end{array}\right\} \tag{10}$$

The authors are working in this new direction and are confident that very soon a very simplified formula [6] can be developed for nuclear binding energy with Up and Down quark masses.

5. Semi-empirical Unified Approach for Fixing the Basic Nuclear Physical Parameters

5.1 Fixing up & down quark rest masses and to fix the strong coupling constant

Step 1: Fixing the Down and Up quark mass ratio

$$\begin{array}{l} \exp\left(\frac{m_dc^2}{m_uc^2}\right)^2 \cong \ln\left(\frac{\hbar c}{Gm_e^2}\right) \\ \rightarrow \frac{m_dc^2}{m_uc^2} \cong \sqrt{\ln\left[\ln\left(\frac{\hbar c}{Gm_e^2}\right)\right]} \cong 2.1529545 \end{array} \tag{11}$$

where G is the gravitational constant. This relation is very complicated to understand and very simple to consider. The authors proposed this relation in 2010 [3]. This can be considered as a hint of final unification. In the published paper [7] and references therein, the authors suggested that,

1) Strength of 'Schwarzschild interaction' can be assumed to be unity.
2) Strength of any other interaction can be defined as the ratio of operating force magnitude and the classical or astrophysical force magnitude $\left(c^4/G\right)$.
3) If one is willing to represent the magnitude of the operating force as a fraction of $\left(c^4/G\right)$ i.e X times of $\left(c^4/G\right)$, where $X << 1$, , then

$$\frac{X \text{ times of } \left(c^4/G\right)}{\left(c^4/G\right)} \cong X \to \text{Effective } G \Rightarrow \frac{G}{X} \tag{12}$$

4) If X is very small, $\frac{1}{X}$ becomes very large. In this way, X can be called as the strength of interaction. Clearly speaking, strength of any interaction is $\frac{1}{X}$ times less than the 'Schwarzschild interaction' and effective G becomes $\frac{G}{X}$.
5) Atomic interaction strength is squared Avogadro number times less than the Schwarzschild interaction and hence atomic gravitational constant can be expressed as:

$$G_A \cong N_A^2 G \tag{13}$$

where N_A is the Avogadro number and is G_A the atomic gravitational constant.

6) Similar to the classical force limit $\left(c^4/G\right)$, in atomic system there exists a characteristic force of magnitude:

$$F_X \cong \left(1/N_A^2\right)\left(c^4/G\right) \cong \left(c^4/N_A^2 G\right) \cong 3.3374\times10^{-4} \text{ N} \tag{14}$$

Based on this proposal, it is possible to show that,

$$\begin{aligned} \hbar &\cong \left(\frac{m_u c^2}{m_d c^2}\right)\sqrt{\left(\frac{G_A m_e^2}{c}\right)\left(\frac{e^2}{4\pi\varepsilon_0 c}\right)} \\ &\cong \left(\frac{m_u c^2}{m_d c^2}\right)\left(m_e c^2\right)\sqrt{\left(\frac{G_A}{c^4}\right)\left(\frac{e^2}{4\pi\varepsilon_0}\right)} \\ &\to \frac{m_d c^2}{m_u c^2} \cong \sqrt{\left(\frac{G_A m_e^2}{\hbar c}\right)\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)} \end{aligned} \tag{15}$$

From relations (11) and (15),

$$\frac{m_d c^2}{m_u c^2} \cong \sqrt{\ln\left[\ln\left(\frac{\hbar c}{G m_e^2}\right)\right]} \cong \sqrt{\left(\frac{G_A m_e^2}{\hbar c}\right)\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)} \tag{16}$$

This is another way of fitting Down and Up quark mass ratio. At this juncture it is really hard to understand the physical meaning of this characteristic relation. With this relation (16) and with reference to the Avogadro number, Newtonian gravitational constant can be fixed like other physical constants. Its magnitude comes out to be $6.673087915\times10^{-11}$ m^{3}kg^{-1}sec^{-2} and $\left(m_d c^2 / m_u c^2\right) \cong 2.152967995$.

Step 2: Fixing the Up and down quark masses

Ratio of Up quark rest mass and electron rest mass can be assumed as follows.

$$\ln\left(\frac{m_u c^2}{m_e c^2}\right) \cong \left(\frac{m_d c^2}{m_u c^2}\right) \cong \sqrt{\ln\left[\ln\left(\frac{\hbar c}{G m_e^2}\right)\right]} \tag{17}$$

$$\begin{aligned} &\rightarrow m_u c^2 \cong \exp\sqrt{\ln\left[\ln\left(\frac{\hbar c}{G m_e^2}\right)\right]}\, m_e c^2 \cong 4.4 \text{ MeV.} \\ &\Rightarrow m_d c^2 \cong \sqrt{\ln\left[\ln\left(\frac{\hbar c}{G m_e^2}\right)\right]}\, m_u c^2 \cong 9.473 \text{ MeV.} \end{aligned} \tag{18}$$

Step 3: Fixing the strong coupling constant and Fine structure ratio

Strong coupling constant and Fine structure ratioboth can be fitted and interrelated as follows.

$$\begin{aligned} &\frac{1}{\alpha_s} \cong \exp\left(\frac{m_d c^2}{m_u c^2}\right) \cong \exp\sqrt{\ln\left[\ln\left(\frac{\hbar c}{G m_e^2}\right)\right]} \\ &\rightarrow \frac{1}{\alpha_s} \cong \exp(2.1529545) \cong 8.61026 \\ &\Rightarrow \alpha_s \cong \left[\exp(2.1529545)\right]^{-1} \cong 0.11614. \end{aligned} \tag{19}$$

With reference to the atomic gravitational constant [7],

$$\frac{1}{\alpha_s} \cong \exp\left(\frac{m_d c^2}{m_u c^2}\right) \cong \exp\sqrt{\left(\frac{G_A m_e^2}{\hbar c}\right)\left(\frac{e^2}{4\pi\varepsilon_0 \hbar c}\right)} \tag{20}$$

$$\rightarrow \frac{1}{\alpha_s} \cong \exp\left(2.15309061\right) \cong 8.611432$$
$$\Rightarrow \alpha_s \cong \left[\exp\left(2.15309061\right)\right]^{-1} \cong 0.116125.$$

If so, relation between strong coupling constant, atomic gravitational constant and down quark mass can be expressed as follows.

$$\frac{1}{\alpha_s} \cong \ln\left(\frac{m_d c^2}{\sqrt{e^2c^4/4\pi\varepsilon_0 G_A}}\right) \cong \ln\sqrt{\frac{4\pi\varepsilon_0 G_A m_d^2}{e^2}} \tag{21}$$

where $\sqrt{e^2c^4/4\pi\varepsilon_0 G_A} \cong \sqrt{\left(\frac{e^2}{4\pi\varepsilon_0}\right)\left(\frac{c^4}{G_A}\right)} \cong 0.001732 \text{ MeV}$ using by which muon and tau masses can be fitted very accurately [7]. Note that $\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_A}} \cong 0.001732 \text{ MeV/c}^2$ can be considered as the characteristic elementary mass unit.

Similarly, relation between strong coupling constant,up quark mass, down quark massand electron mass can be expressed as follows.

$$\begin{aligned} m_u c^2 &\cong \left(\frac{1}{\alpha_s}\right) m_e c^2 \cong 4.4 \text{ MeV} \\ \rightarrow \frac{m_u c^2}{m_e c^2} &\cong \left(\frac{1}{\alpha_s}\right) \cong \exp\left(\frac{m_d c^2}{m_u c^2}\right) \end{aligned} \tag{22}$$

Now relation (15) can be re-expressed as follows.

$$\begin{aligned} \hbar c &\cong \alpha_s \left(\frac{m_u^2 c^4}{m_d c^2}\right)\sqrt{\left(\frac{G_A}{c^4}\right)\left(\frac{e^2}{4\pi\varepsilon_0}\right)} \\ \rightarrow \frac{1}{\alpha} &\cong \alpha_s \left(\frac{m_u}{m_d}\right)\sqrt{\frac{4\pi\varepsilon_0 G_A m_u^2}{e^2}} \end{aligned} \tag{23}$$

5.2 On the neutron rest mass and neutron–proton mass difference

Qualitatively and quantitatively neutron rest energy can be interlinked with the proposed up and down quarks as expressed in the following way.

$$m_n c^2 \cong \left(\frac{1}{\alpha}+\frac{1}{\alpha_s}\right)\sqrt{m_u m_d}\,c^2 \cong \left(\frac{\alpha_s+\alpha}{\alpha_s \alpha}\right)\sqrt{m_u m_d}\,c^2 \cong 940.27 \text{ MeV}$$
$$\rightarrow \frac{m_n c^2}{\sqrt{m_u m_d}\,c^2} \cong \left(\frac{1}{\alpha}+\frac{1}{\alpha_s}\right) \cong \left(\frac{\alpha_s+\alpha}{\alpha_s \alpha}\right) \tag{24}$$

By considering this relation (24), strong coupling constant can also be fitted. Neutron and proton rest energy difference can be fitted in the following way.

$$\frac{\sqrt{m_u m_d}\,c^2}{m_n c^2 - m_p c^2} \cong \ln\left(\frac{1}{\alpha}+\frac{1}{\alpha_s}\right) \cong \ln\left(\frac{\alpha_s+\alpha}{\alpha_s \alpha}\right)$$
$$\rightarrow m_n c^2 - m_p c^2 \cong \left[\ln\left(\frac{1}{\alpha}+\frac{1}{\alpha_s}\right)\right]^{-1}\sqrt{m_u m_d}\,c^2 \cong 1.296 \text{ MeV} \tag{25}$$

5.3 On the Semi empirical mass formula energy coefficients

The semi empirical mass formula energy coefficients can be fitted and understood as follows. For understanding purpose, data fitting purpose and for simplicity, $\sqrt{m_u m_d}\,c^2$ is taken as a reference. Interested researchers may consider any other characteristic mass unit that connected with up and down quark masses.

Step 1: Volume energy coefficient

It can be fitted as follows.

$$a_v \cong \ln\sqrt{\frac{m_p c^2}{\sqrt{m_u m_d}\,c^2}} \cdot \sqrt{m_u m_d}\,c^2 \cong 16.07 \text{ MeV} \cong 16.1 \text{ MeV} \tag{26}$$

where m_p is the rest mass of proton. Thus it can be suggested that,

$$\frac{a_v}{\sqrt{m_u m_d}\,c^2} \cong \ln\sqrt{\frac{m_p c^2}{\sqrt{m_u m_d}\,c^2}} \tag{27}$$

Step 2: Surface energy coefficient

It can be fitted as follows.

$$a_s \cong \sqrt{\frac{1}{\alpha_s}} \cdot \sqrt{m_u m_d}\,c^2 \cong 18.94 \text{ MeV} \tag{28}$$

Thus it can be suggested that,

$$\frac{a_s}{\sqrt{m_u m_d}\,c^2} \cong \sqrt{\frac{1}{\alpha_s}} \tag{29}$$

Step 3: Coulombic energy coefficient

It can be fitted as follows.

$$a_c \cong \alpha_s \sqrt{m_u m_d}\,c^2 \cong 0.75 \text{ MeV} \tag{30}$$

Thus it can be suggested that,

$$\frac{a_c}{\sqrt{m_u m_d}\,c^2} \cong \alpha_s \tag{31}$$

With this Coulombic energy coefficient 'ending of strong interaction range' or 'beginning of coulombic repulsion range' can be expressed as follows.

$$R_s \cong \frac{e^2}{4\pi\varepsilon_0 a_c} \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 \sqrt{m_u m_d}\,c^2} \cong 1.92 \text{ fm} \tag{32}$$

With this relation (32) it is noticed that,

$$\left.\begin{aligned} &R_u \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 m_u c^2} \cong 2.818 \text{ fm}, \\ &R_d \cong \left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 m_d c^2} \cong 1.309 \text{ fm} \\ &\sqrt{R_u R_d} \cong 1.92 \text{ fm},\ \frac{R_u^2}{R_d} \cong 0.61 \text{ fm}, \\ &\frac{2R_u^2}{R_d} \cong 1.216 \text{ fm and } \frac{\sqrt{2}R_u^2}{R_d} \cong 0.86 \text{ fm} \end{aligned}\right\} \tag{33}$$

Here, 1.216 fm can be compared with the characteristic 'nuclear charge radius' and 0.86 fm can be compared with the 'RMS radius' of proton. With $\left(R_u^2/R_d\right) \cong 0.61$ fm, RMS charge radii of atomic nuclei [8] can be fitted as follows.

$$\left.\begin{aligned} R_{(N,Z)} &\cong \left[(Z+2)+(N+2)\right]^{\frac{1}{3}}\left(R_{up}^2/R_{down}\right) \\ &\cong \left[(Z+2)+(N+2)\right]^{\frac{1}{3}} 0.61 \text{ fm}, \quad \text{where } N = A - Z \end{aligned}\right\} \tag{34}$$

Step 4: Asymmetry energy coefficient

Asymmetryenergy coefficient can be fitted as follows.

$$a_a \cong \left[\left(\frac{m_d}{m_u}\right)^2 - 1\right]\sqrt{m_u m_d}\,c^2 \cong 23.47 \text{ MeV} \tag{35}$$

Thus it can be suggested that,

$$\frac{a_a}{\sqrt{m_u m_d}\,c^2} \cong \left[\left(\frac{m_d}{m_u}\right)^2 - 1\right] \tag{36}$$

Step 5: Pairing energy coefficient

Pairing energy coefficient can be fitted as follows.

$$a_p \cong \frac{1}{2}\left[\left(\frac{m_d}{m_u}\right)^2 - 1\right]\sqrt{m_u m_d}\,c^2 \cong 11.735 \text{ MeV} \tag{37}$$

Thus it can be suggested that,

$$\frac{a_p}{\sqrt{m_u m_d}\,c^2} \cong \frac{1}{2}\left[\left(\frac{m_d}{m_u}\right)^2 - 1\right] \tag{38}$$

See table-3 for the comparison of current and proposed energy coefficients. See table-4 for the measured and estimated binding energy from SEMF with the proposed energy coefficients.

Table 3. Current and proposed SEMF binding energy coefficients

	a_v MeV	a_s MeV	a_c MeV	a_a MeV	a_p MeV
Current	15.78	18.34	0.71	23.21	12.0
Proposed	16.07	18.944	0.75	23.47	11.735

Table 4. Fitting the measured binding energy with the proposed SEMF energy coefficients

Z	A	**Measured** $(BE)_{meas}$ **in MeV**	**Calculated** $(BE)_{cal}$ **in MeV**
26	56	492.258	490.07
44	100	861.928	860.53
50	116	988.684	986.0
70	170	1378.13	1373.28
82	208	1636.43	1619.12
92	238	1801.69	1797.12

5.4 To understand the magic numbers

Considering the strong interaction that takes place in between Up-Up, Up-Down and Down-Down along with the neutron-proton as a whole, one can expect four kinds of 'strong attractive' forces within the nucleus. Considering the number 'four' as a characteristic structural feature, observed magic numbers [10] can be fitted in the following.

Step 1: The magic plateaus'

$$P_I \cong 4\left[n(n+1)\right]+4 \cong 4I+4 \tag{39}$$

where $n=0,1,2,3,..$ and $I=n(n+1)$.

Magic plateau begins at $(P_I)_{start} \cong 4\left[n(n+1)\right]$ and extends up to $(P_I)_{end} \cong 4\left[n(n+1)\right]+4$.

Step 2: Mid of the magic plateaus

It is noticed that, mid of the magic plateau is very close to the observed magic numbers.

$$M \cong \frac{(P_I)_{start}+(P_I)_{end}}{2} \tag{40}$$

Step 3: Mid of the ending of consecutive magic plateaus'

Mid of the ending of consecutive magic plateaus' seems to be close to the observed semi magic numbers.

$$M \cong \frac{(P_{I1})_{end}+(P_{I2})_{end}}{2} \tag{41}$$

where $I1=n(n+1)$ and $I2=(n+1)(n+2)$. See the following table-5 for the comparison of actual and proposed magic numbers. ***From the table-5, it is very clear to say that, proposed magic plateau mid is close to and proportional to the corresponding magic numbers***. Magic numbers like 114,126, 164,174,..etc are found to be missing from the net and authors are working on this.

Table 5. Fitting the magic numbers

n	**I**	**4I**	**4I+4**	**[(4I)+(4I+4)/2**	**Actual magic numbers**	**Mid of the ending of consecutive magic plateaus'**
0	0	0	4	2	2	
1	2	8	12	10	8	***(4+12)/2=8***
2	6	24	28	26	20, 28	***(12+28)/2=20***
3	12	48	52	50	50	***(28+52)/2=40***
4	20	80	84	82	82	(52+84)/2=68
5	30	120	124	122	(114,122,124,126)	(84+124)/2=104

6	42	168	172	170	(164,174,184,196)	(124+172)/2=148
7	56	224	228	226	(236)	(172+228)/2=200
8	72	288	292	290	(318)	(228+292)/2=260

6. Conclusion

In consideration of all of the above proposed relations, concepts and data fitting, a scale independent 'quark sea' model of observed and stable atomic nuclei can be developed with confidence in the near future. The five energy coefficients of the famous semi-empirical mass formula can be understood in terms of up and down quarks.

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References

[1] Denis Lacroix. Introduction - Strong interaction in the nuclear medium: new trends. International Joliot Curie School (EJC2009). http://www.cenbg.in2p3.fr/heberge/EcoleJoliotCurie/coursannee/cours/D_lacroix.pdf
[2] A.V. Manohar and C.T. Sachrajda. Quark masses. Updated Jan 2014. Pages 1-20. (pdg.lbl.gov/2014/reviews/rpp2014-rev-quark-masses.pdf)
[3] U. V. S. Seshavatharam and S. Lakshminarayana. Super Symmetry in Strong and Weak interactions. Int. J. Mod. Phys. E, Vol.19, No.2, p.263-280 (2010).
[4] Chowdhury, P.R. et al. Modified Bethe-Weizsacker mass formula with isotonic shift and new drip lines. Mod. Phys. Lett. A20 p.1605-1618. (2005)
[5] K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014)
[6] U. V. S. Seshavatharam and S. Lakshminarayana. On the role of RMS radius of proton in understanding nuclear binding energy. Journal of Applied Physical Science International, Vol.: 2, Issue. 3 , p84-100, 2015.
[7] U. V. S. Seshavatharam and S. Lakshminarayana. On fixing the magnitudes of gravitational constant and strong coupling constant. International Journal of Advanced Astronomy, 3 (1) 17-23, (2015)
[8] U. V. S. Seshavatharam and S. Lakshminarayana. Consideration on a Nuclear Binding Energy Formula. Prespacetime Journal, Volume 6, Issue 1, pp. 58-75. January 2015.
[9] P. Marmier and E. Sheldon, Physics of Nuclei and Particles. Academic Press, New York,1969, vol.II.
[10] Gabriel Mart´ınezPinedo. Shell-Model Applications in nuclear physics and astrophysics. 10th Euro Summer School on Exotic Beams. page 81. (2003). http://ific.uv.es/~euschool/lec_martinez-pinedo.pdf