Article

LRS Bianchi Type II Tilted Stiff Fluid Model with Heat Conduction in General Relativity

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Abstract

Locally Rotationally Symmetric (LRS) Bianchi Type II tilted stiff fluid model with heat conduction in General Relativity is investigated. To get the deterministic solution in terms of cosmic time t, we have assumed the stiff fluid condition (ρ=p), ρ being matter density p the isotropic pressure and a supplementary condition $R = S^n$ between metric potentials R and S where n is a constant. The model starts with a big bang at $T=1/(\sqrt{\beta})$ and the expansion decreases as time increases. The model in general represent non-tilted one. Since the deceleration parameter $q > 0$, hence the model represents decelerating phase of universe. The model has point type singularity at $T = 0$.

Keywords: LRS Bianchi II, tilted, stiff fluid, heat conduction.

1. Introduction

Stiff fluid universes create more interest in the study because for these models, the speed of light is equal to the speed of sound and its governing equations have the same characteristics as those of Einstein's field equations (Zel'dovich [1]). Stiff fluid models are investigated by the barotropic equation of state $p = \gamma\rho$, $0 \le \gamma \le 1$, ρ the matter density, p the isotropic pressure when $\gamma = 1$. Thus, we have $\rho = p$. Barrow [2] in his investigation has shown that entropy level of universe makes it likely that its initial state was isotropic and quiescent ($p = \gamma\rho$, $\gamma \in (-1,0)$ rather than chaotic only if the equation of state is stiff fluid i.e. $\rho = p$. Keeping in view the importance of stiff fluid models, Bali et al. [3,4], Mak and Harko [5] have investigated cosmological models for stiff fluid distribution in different contexts.

There has been a considerable interest in the study of homogeneous and anisotropoic models in which the fluid flow is not normal to the hypersurface of homogeneity. These models are called Tilted cosmological models. Kaisner [6] in his investigation has indicated that observations of large scale streaming of matter (Lynden-Bell et al. [7]) create more interest in the study due to their relevance regarding theories of structure formation. Tilting cosmological models have been

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advanced to study the effect of a large scale peculiar velocity relation to CMB (Cosmic Microwave Background) frame (Coley [8]), Coley and Tupper [9]) particularly the growth of inhomogeneities and their relationship with the observed large scale structure. In particular, these models have relevance in the study of models which are linear perturbation of FRW models. Bradley and Sviestins [10] have pointed out that a tilted fluid experiences an inhomogeneous energy density in its rest frame (which does not coincide with the surface of homogeneity) so that it is more natural for the field to respond by conducting heat than for it to retain the perfect fluid form.

The general dynamics of tilted cosmological models are given by Ellis and King [11], King and Ellis [12], Mukherjee [13] investigated Bianchi Type I cosmological model with heat flux for perfect fluid distribution in General Relativity. The cosmological models with heat flux have been studied by number of authors viz. Novello and Reboucas [14], Roy and Banerjee [15], Coley and Tupper [16], Banerjee and Santos [17], Coley [18], Bali et al. [19.20.21].

2. Metric and Field Equations

We consider LRS (Locally Rotationally Symmetric) Bianchi Type II space-time as

$$ds^2 = -dt^2 + R^2(dx^2 + dz^2) + S^2(dy - x\,dz)^2 \tag{1}$$

The energy-momentum tensor for perfect fluid distribution with heat conduction is given by Ellis [22] as

$$T_i^j = (\rho + p)v_i v^j + p g_i^j + q_i v^j + v_i q^j \tag{2}$$

together with

$$g_{ij}\, v^i v^j = -1 \tag{3}$$

$$q_i\, q^i > 0 \tag{4}$$

$$q_i\, v^i = 0 \tag{5}$$

where ρ is the matter density, p the isotropic pressure, v^i the fluid flow vector having the components $\left(\frac{\sinh\lambda}{R}, 0, 0, \cosh\lambda\right)$ satisfying (3) and λ is the tilt angle.

The Einstein's field equation

$$R_i^j - \frac{1}{2} R\, g_i^j = -8\pi T_i^j, \quad \Lambda = 0 \tag{6}$$

(in geometrized units G = 1, c = 1) for the line-element (1) leads to

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{R_4 S_4}{RS} + \frac{S^2}{4R^4} = -8\pi[(\rho + p)\sinh^2\lambda + p + 2Rq^1 \sinh\lambda] \tag{7}$$

$$\frac{2R_{44}}{R} + \frac{R_4^2}{R^2} - \frac{3S^2}{4R^4} = -8\pi p \tag{8}$$

$$\frac{2R_4 S_4}{RS} + \frac{R_4}{R^2} - \frac{S^2}{4R^4} = -8\pi[-(\rho + p)\cosh^2\lambda + p - 2Rq^1 \sinh\lambda] \tag{9}$$

$$(\rho + p) R \sinh\lambda \cosh\lambda + R^2 q^1 \frac{\cosh 2\lambda}{\cosh\lambda} = 0 \tag{10}$$

where subscript 4 denotes the differentiation with respect to t.

3. Solution of Field Equations

For the complete determination of the set of equations, we assume that the universe is filled with stiff fluid distribution i.e.

$$\rho = p \tag{11}$$

and the condition between metric potentials Rand S as

$$R = S^n \tag{12}$$

where n is constant.

From. equation (7), (9) and (11), we have

$$\frac{R_{44}}{R} + \frac{S_{44}}{S} + \frac{3R_4 S_4}{RS} + \frac{R_4^2}{R^2} = 0 \tag{13}$$

Equations (12) and (13) lead to

$$\frac{S_{44}}{S} + \left\{ \frac{(3n^2 - 2n) + (4n - n^2)}{2n + (1 - n)} \right\} \frac{S_4^2}{S^2} = 0 \tag{14}$$

To get determinate solution in terms of cosmic time t, we assume that n = 1/2. Thus equation (14) leads to

$$\frac{S_{44}}{S_4} = -\frac{S_4}{S} \tag{15}$$

which leads to

$$\log S_4 = \log \alpha - \log S$$

Thus, we have

$$S_4 = \frac{\alpha}{S}. \tag{16}$$

From (16), we have

$$SdS = \alpha dt$$

$$S^2 = 2\alpha t + b \tag{17}$$

where $a = 2\alpha = (at+b)$

Therefore

$$R^2 = S = (at+b)^{1/2} \tag{18}$$

Now the metric (1) leads to

$$ds^2 = -dt^2 + (at+b)^{1/2}(dx^2 + dz^2) + (at+b)(dy - xdz)^2 \tag{19}$$

$$= -\frac{dT^2}{a^2} + T^{1/2}(dx^2 + dz^2) + T(dy - x\,dz)^2 \tag{20}$$

4. Some Physical and Geometrical Features

We have

$$S^2 = at + b$$

$$\frac{S_4}{S} = \frac{a}{2(at+b)}$$

The matter density (ρ), the isotropic pressure (p), the tilt angle λ, components of flow vector v^1 and v^4 of v^i, the components of shear tensor σ_{11} and σ_{14} by

$$8\pi\rho = \frac{5a}{16(at+b)^2} + \frac{3}{4} = 8\pi p$$

$$=\frac{5a}{16T^2}+\frac{3}{4}=8\pi p \tag{21}$$

$$\cosh^2\lambda=\frac{1+\beta(at+b)^2}{1-\beta(at+b)^2}=\frac{1+\beta T^2}{1-\beta T^2} \tag{22}$$

where

$$\beta=\frac{4}{5a} \tag{23}$$

$$\theta=\frac{a[4\beta(at+b)^2+2\{1-\beta^2(at+b)^4\}]}{2(at+b)\{1-\beta(at+b)^2\}^{3/2}\{1+\beta(at+b)^2\}^{1/2}}$$

$$=\frac{a\,[4\beta T^2+2\{1-\beta^2T^4]}{2T(1-\beta T^2)^{3/2}(1+\beta T^2)^{1/2}} \tag{24}$$

$$\sigma_{11}=\frac{a\{1+\beta(at+b)^2\}^{1/2}\left[-\frac{1}{2}\{1-\beta^2(at+b)^4\}+8\beta(at+b)^2\right]}{3\{2(at+b)\}^{1/2}\{1-\beta(at+b)^2\}^{5/2}} \tag{25}$$

$$\sigma_{14}=\frac{-\sqrt{2\beta}\,a\left[\left(-\frac{1}{2}\right)\{1-\beta^2(at+b)^4\}+8\beta(at+b)^2\right]}{3(2)^{3/4}(at+b)^{-1/4}\{1-\beta(at+b)^2\}^{5/2}} \tag{26}$$

$$v^1=\frac{\sqrt{2\beta}\,(at+b)^{3/4}}{(2)^{1/2}\{1-\beta(at+b)^2\}^{1/2}}$$

$$=\frac{\sqrt{2\beta}\,T^{3/4}}{(2)^{1/2}(1-\beta T^2)^{1/2}} \tag{27}$$

$$v^4=\frac{\{1+\beta(at+b)^2\}^{1/2}}{\{1-\beta(at+b)^2\}^{1/2}}=\frac{(1+\beta T^2)^{1/2}}{(1-\beta T^2)^{1/2}} \tag{28}$$

$$V=R^2S=2(at+b)=2T \tag{29}$$

$$q=-\frac{\ddot{V}/V}{\dot{V}^2/V^2}=2 \tag{30}$$

The shear tensor (σ_{ij}) and heat conduction vector q^i satisfy trace free conditions

$$\sigma_{ij}\,v^j=0 \tag{31}$$

and

$$q_i\,v^i=0 \tag{32}$$

5. Discussion and Conclusion

The reality condition $\rho > 0$ for the model (20) leads to

$$\frac{5a}{16T^2} + \frac{3}{4} > 0$$

where $a > 0$.

The model starts with a big-bang at $T = \frac{1}{\sqrt{\beta}}$ and the expansion in the model decreases as time increases. The model in general represents tilted model for perfect fluid distribution. However, if $\beta = 0$, then the model leads to non-tilted one because in this case, the tilt angle $\lambda = 0$. The spatial volume increases with time. Since the deceleration parameter $q > 0$, hence the model represents decelerating phase of the universe. The shear tensor (σ_{ij}) and heat conduction vector (q_i) satisfy trace free condition

$$\sigma_{ij} v^j = 0$$

and

$$q_i \, v^i = 0$$

There is a Point Type singularity in the model at T = 0(MacCallum [23]).

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References

[1] Zel'dovich, YaB 1970 Mon. Not. Roy. Astron. Soc. 160
[2] Barrow J D 1978 Nature **272** 211
[3] Bali R and Sharma K 2003Astropohys. Space-Science **283** 11
[4] Bali R Ali M and Jain VC 2004 Int. J. Theor. Phys. **47**, 2218
[5] Mak M K and Harko T 2004 Int. J. Mod. POhys. D**13** 273
[6] Kaiser N 1991 Astropohysical J. **366** 388
[7] Lynden-Bell D et al. 1991 Astrophysical J. **326** 388
[8] Coley A A 1987 Astropohysical J. **318** 487
[9] Colely A A and Tupper B O J 1988 J. Maths. Phys. **27** 406
[10] Bradley J M and Sviestins E 1984 Gen. Relativ. Gravt. **16** 1119
[11] Ellis G F R and King A R 1974 Comm. Math. Phys. **38** 119
[12] King A R and Ellis G F R 1973 Comm. Math. Phys. **31** 209
[13] Mukherjee G 1983 J. Astrophys. and Astronomy **4** 295
[14] Novello M and Reboucas M J 1978 Astropohys. J. **225** 719

[15] Roy S R and Banerjee S K 1988 Astrophys. and Space-Science **150** 213
[16] Coley A A and Tupper B O J 1983 Phys. Lett. A **95** 357
[17] Banerjee A A and Santos N O 1986 Gen. Relativ. and Grav. **18** 1251
[18] Coley A A 1990 Gen. Relativ. Gravt. **22** 3
[19] Bali R and Sharma K 2002 Pramana – J. Phys. **58** 457
[20] Bali R and Kumawat P 2008 Gravt. and Cosmology **14** 347
[21] Bali R and Kumawat P 2010 J. Phys. **40** 1
[22] Ellis G F R 1971 General Relativity and Cosmology ed. R.K. Sachs, Academic Press, p.116
[23] MacCallum M A H 1971 Communication in Math.Phys. **20** 57