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Bianchi Type-VI$_0$ with Strange Quark Matter Attached to String Cloud in Saez-Ballester Theory of Gravitation

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Abstract

Bianchi type-VI$_0$ with strange quark matter attached to a string cloud in Saez- Ballester (1986) theory of gravitation has been studied. The field equations have been solved by using the anisotropy feature of the universe in Bianchi type-VI$_0$ space time. Some important features of the model, thus obtained, have been discussed.

Keywords: Bianchi type-VI$_0$, space-time, quark matter, cosmic strings, Saez-Ballester theory of gravitation.

1. Introduction

Saez and Ballester (1986) developed a theory in which the metric is coupled with a dimensionless scalar field. This coupling gives a satisfactory description of weak fields. This theory also suggests a possible way to solve missing matter problem in non flat FRW cosmologies. The field equations given by Saez- Ballester (1986) for the combined scalar and tensor fields (using geometrized units with $c = 1$, $8\pi G = 1$) are

$$R_{ij} - \frac{1}{2} R g_{ij} - \omega \phi^{m} \left(\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) = -T_{ij} \tag{1.1}$$

with the scalar field ϕ satisfies the equation

$$2\phi^{m} \phi^{,i}{}_{;i} + m\phi^{m-1} \phi_{,k} \phi^{,k} = 0 \tag{1.2}$$

and $T^{ij}{}_{;j} = 0$ (1.3)

where $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$ is an Einstein tensor, R is the scalar curvature, ω and m are arbitrary constants.

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The study of cosmological models in the framework of scalar –tensor theories has been the active area of research for the last few decades.Saez (1987) has discussed the initial singularity and inflationary universe in this theory. He has also obtained non-singular FRW model for k=0. Singh and Agrawal (1991), Shri Ram and Tiwari (1998), Reddy and Naidu (2007) and Rao et al. (2007) are some of the authors who have investigated several aspects of the cosmological models in this theory. Rao et al. (2011) have discussed anisotropic universe with cosmic strings and bulk viscosity and very recently Rao et al. (2012) have studied Bianchi type- I dark energy model in Saez – Ballester scalar tensor theory of gravitation.

In this study, we will attach strange quark matter to the string cloud. It is plausible to attach strange quark matter to the string cloud. Strange quark matter is modelled with an equation of state based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are through as degenerate Fermi gas, which exists only in a region of space endowed with a vacuum energy density B_c (called as the bag constant). In the framework of this model, the quark matter is composed of mass less *u* and *d* quarks, massive *s* quarks and electrons. In the simplified version of the bag model, it is assumed that quarks are mass less and non interacting.

Therefore, we have quark pressure

$$p_q = \frac{\rho_q}{3} \tag{1.4}$$

where ρ_q is the quark energy density.

The total energy density is

$$\rho = \rho_q + B_C \tag{1.5}$$

and the total pressure is

$$p = p_q - B_C \tag{1.6}$$

For more information and review of strange quark matter attached to string cloud models one can refer Adhav et al. (2009). Khadekar et al. (2009) have confined their work to the quark matter attached to the topological defects in general relativity. Khadekar and Rupali Wanjari (2012) have discussed geometry of quark and strange quark matter in higher dimensional general relativity. Rao and Neelima (2012) have discussed axially symmetric space time with strange quark matter attached to the string cloud in self creation theory and general relativity and established that the additional condition, special law of variation of Hubble parameter proposed by Bermann (1983), taken by Katore and Shaikh (2012) in general relativity is superfluous. Recently, Rao and Sireesha (2012) have discussed axially symmetric space time with strange quark matter attached to the string cloud Brans-Dicke theory of gravitation.

In this paper we will study Bianchi type-VI_0 with strange quark matter attached to a string cloud in Saez- Ballester theory of gravitation.

2. Metric and Energy Momentum Tensor

We consider the Bianchi –VI_0 line element in the form

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 e^{-2x} dz^2 \tag{2.1}$$

where $A, B\,\&\,C$ are the functions of time ‘t’ only.

The energy momentum tensor for string cloud (Letelier 1983) is given by

$$T_{ij} = \rho\, u_i\, u_j - \rho_S x_i x_j \tag{2.2}$$

Here ρ is the rest energy density for the cloud of strings with particles attached to them and ρ_S is the string tension density. They are related by

$$\rho = \rho_p + \rho_S \tag{2.3}$$

where ρ_p is the particle energy density.

We know that string is free to vibrate. The vibration models of the string represent different types of particles because these models are seen as different masses or spins. Therefore, here we will take quarks instead of particles in the string cloud. Hence we consider quark matter energy density instead of particle energy density in the string cloud.

In this case from (2.3), we get

$$\rho = \rho_q + \rho_S + B_c \tag{2.4}$$

From (2.3) & (2.4), we have energy momentum tensor for strange quark matter attached to the string cloud (Yavuz et al. 2005) as

$$T_{ij} = (\rho_q + \rho_S + B_c) u_i u_j - \rho_S x_i x_j \tag{2.5}$$

where u_i is the four velocity of the particles and x_i is the unit space like vector representing the direction of string.

We have u_i and x_i with

$$u_i u^i = -x_i x^i = 1 \; and \; u^i x_i = 0 \tag{2.6}$$

We have taken the direction of string along x-axes. Then the components of energy momentum tensor are

$$T_1^1 = \rho_S, T_2^2 = T_3^3 = 0, T_4^4 = \rho \tag{2.7}$$

where ρ and ρ_s are functions of 't' only.

3. Solution of the Field equations:

The field equations for the metric (2.1) can be written as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} - \frac{\omega}{2}\phi^m \dot{\phi}^2 = \rho_S \tag{3.1}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{1}{A^2} - \frac{\omega}{2}\phi^m \dot{\phi}^2 = 0 \tag{3.2}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}-\frac{\omega}{2}\phi^m\dot{\phi}^2=0 \tag{3.3}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{1}{A^2}+\frac{\omega}{2}\phi^m\dot{\phi}^2=\rho \tag{3.4}$$

$$\frac{\dot{C}}{C}-\frac{\dot{B}}{B}=0 \tag{3.5}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+\frac{m}{2}\frac{\dot{\phi}^2}{\phi}=0 \tag{3.6}$$

$$\dot{\rho}+\rho\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-\rho_s\left(\frac{\dot{A}}{A}\right)=0 \tag{3.7}$$

Here the over head dot denotes differentiation with respect to‘t’.

From (3.5), we get

$$C=\alpha B$$

Without loss of generality we can take $\alpha=1$, so that we have

$$C=B \tag{3.8}$$

Using (3.8), the field equations (3.1) to (3.7) reduce to

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{1}{A^2}-\frac{\omega}{2}\phi^m\dot{\phi}^2=\rho_s \tag{3.9}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}-\frac{\omega}{2}\phi^m\dot{\phi}^2=0 \tag{3.10}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}-\frac{1}{A^2}+\frac{\omega}{2}\phi^m\dot{\phi}^2=\rho \tag{3.11}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)+\frac{m}{2}\frac{\dot{\phi}^2}{\phi}=0 \tag{3.12}$$

$$\dot{\rho}+\rho\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)-\rho_s\left(\frac{\dot{A}}{A}\right)=0 \tag{3.13}$$

By taking the transformation $dt = AB^2\, dT$ **,** the above field equations can be written as

$$2\frac{B''}{B} - 3\frac{B'^2}{B^2} - 2\frac{A'B'}{AB} + B^4 - \frac{\omega}{2}\phi^m \phi'^2 = \rho_s (A^2 B^4) \tag{3.14}$$

$$\frac{A''}{A} - \frac{A'^2}{A^2} + \frac{B''}{B} - 2\frac{B'^2}{B^2} - 2\frac{A'B'}{AB} - B^4 - \frac{\omega}{2}\phi^m \phi'^2 = 0 \tag{3.15}$$

$$2\frac{A'B'}{AB} + \frac{B'^2}{B^2} - B^4 + \frac{\omega}{2}\phi^m \phi'^2 = \rho (A^2 B^4) \tag{3.16}$$

$$\phi'' + \frac{m}{2}\frac{\phi'^2}{\phi} = 0 \tag{3.17}$$

$$\rho' + \rho\left(\frac{A'}{A} + 2\frac{B'}{B}\right) - \rho_s\left(\frac{A'}{A}\right) = 0 \tag{3.18}$$

From here after the over head dash denotes differentiation with respect to ‘T’.

The field equations (3.14) to (3.17) are only four independent equations with five unknowns A, B, ρ, ρ_s and ϕ. Since these equations are highly non-linear in nature, in order to get a deterministic solution we take the following plausible physical condition, the shear scalar σ is proportional to scalar expansion θ, which leads to the linear relationship between the metric potentials A and B, i.e.,

$$A = B^n \tag{3.19}$$

where n is an arbitrary constant.

From (3.17), we get

$$\phi = \left[\left(\frac{m+2}{2}\right)(k_1 T + k_2)\right]^{\frac{2}{m+2}} \quad , \tag{3.20}$$

Using (3.19) & (3.20) in (3.15), we get

$$\frac{B''}{B}-\frac{(2+3n)}{(n+1)}\frac{B'^2}{B^2}=\frac{B^4}{(n+1)}+\frac{\omega k_1^{\;2}}{2(n+1)},n\neq-1. \tag{3.21}$$

Let $B'=f(B)$,

then from (3.21), we get

$$\frac{d}{dB}(f^2)-\frac{2(3n+2)}{(n+1)B}f^2=\frac{2}{(n+1)}B^5+\frac{\omega k_1^{\;2}}{(n+1)}B$$

From the above equation, we get

$$f(B)=B\sqrt{B^4-k_3^{\;2}}\quad, k_3^{\;2}=\frac{\omega k_1^{\;2}}{2(2n+1)}\neq 0, \tag{3.22}$$

From (3.22), we get

$$B=[k_3\sec(2k_3T)]^{\frac{1}{2}} \tag{3.23}$$

From (3.19), we get

$$A=[k_3\sec(2k_3T)]^{\frac{n}{2}} \tag{3.24}$$

From (3.8), we get

$$C=[k_3\sec(2k_3T)]^{\frac{1}{2}} \tag{3.25}$$

The metric (2.1) can now be written as

$$\begin{aligned} ds^2 &= [k_3\sec(2k_3T)]^{n+2}dT^2-[k_3\sec(2k_3T)]^n dx^2 \\ &\quad -[k_3\sec(2k_3T)]e^{2x}dy^2-[k_3\sec(2k_3T)]e^{-2x}dz^2 \end{aligned} \tag{3.26}$$

From (3.16), we get string energy density

$$\rho = \frac{2n}{k_3{}^n} \cos^n (2k_3 T) \tag{3.27}$$

From (3.14), we get string tension density

$$\rho_s = \frac{2(2-n)}{k_3{}^n} \cos^n (2k_3 T) \tag{3.28}$$

String particle density

$$\rho_p = \rho - \rho_s = \frac{4(n-1)}{k_3{}^n} \cos^n (2k_3 T) \tag{3.29}$$

Quark energy density

$$\rho_q = \rho - B_c = \frac{2n}{k_3{}^n} \cos^n (2k_3 T) - B_c \tag{3.30}$$

Quark pressure

$$p_q = \frac{\rho_q}{3} = \frac{2n}{3k_3{}^n} \cos^n (2k_3 T) - \frac{B_c}{3} \tag{3.31}$$

Hence the metric (3.26) together with (3.20) & (3.27) to (3.31) represents singularity free Bianchi type-VI$_0$ with strange quark matter attached to string cloud in Saez - Ballester theory of gravitation.

4. Some important features of the model:

The Volume element of the model (3.26) is given by

$$V = (-g)^{1/2} = \left[k_3 \sec(2k_3 T)\right]^{\frac{n+2}{2}} \tag{4.1}$$

The expression for the expansion scalar θ is given by

$$\theta = u^i;_i = \frac{(n+2)}{k_3^{\frac{n}{2}}} \sin(2k_3T)\cos^{\frac{n}{2}}(2k_3T) \tag{4.2}$$

and the shear σ is given by

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{7}{18}\frac{(n+2)^2}{k_3^{\,n}}\sin^2(2k_3T)\cos^n(2k_3T) \tag{4.3}$$

The deceleration parameter q is given by

$$q = -3\theta^{-2}(\theta_{,i}u^i + \frac{1}{3}\theta^2)$$

$$= -\left(\frac{3k_3^{\frac{n}{2}}}{(n+2)}\left[-nk_3[\sec(2k_3T)]^{\frac{n+2}{2}} + 2k_3\cos ec^2(2k_3T)[\sec(2k_3T)]^{\frac{n-2}{2}}\right]+1\right) \tag{4.4}$$

Here q→-1 as T→∞ for even values of n.

The Hubble parameter (H) is given by

$$H = \frac{(n+2)}{3k_3^{\frac{n}{2}}}\sin(2k_3T)\cos^{\frac{n}{2}}(2k_3T) \tag{4.5}$$

The average anisotropy parameter (A_m)is given by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = \frac{2(n-1)^2}{(n+2)^2}, \quad n \neq -2 \tag{4.6}$$

where $\Delta H_i = H_i - H \ \ (i = 1,2,3)$.

The density parameter Ω is given by

$$\Omega = \frac{\rho}{3H^2} = \frac{6n}{(n+2)^2}\cos ec^2(2k_3T) \tag{4.7}$$

The tensor of rotation

$w_{ij} = u_{i,j} - u_{j,i}$ is identically zero and hence this universe is non-rotational.

5. Conclusions

It is observed that the model (3.26) has singularities at $T = \frac{(2r+1)\pi}{2k_3}, r = 0, \pm 1, \pm 2, \ldots$ and as $T \to \infty$, the proper volume will vanish. For $n = 2$, we get dust quark matter solution, i.e. $\rho_s = 0$ *and* $\rho_q + B_c = \rho_p = \rho = \frac{40}{\omega k_1^2} \cos^2(2k_3 T)$. Since s $T \to \infty, q \to -1$ for even values of n, the present model represents an accelerating universe. Also for $n = 1$ the mean anisotropy parameter is zero. Hence the model (3.26) will become isotropic for $n = 1$. From (3.29), we can see that the particle density disappears and we will get only isotropic geometric string model for $n = 1$ i.e. $\rho_p = 0$ and $\rho = \rho_s = \frac{2}{k_3} \cos(2k_3 T)$. For our model (3.26), $\frac{\sigma}{\theta} \approx 0.6236$ which is greater than present upper limit $(10)^{-5}$ of $\frac{\sigma}{\theta}$ obtained by Collins et al. (1980) from indirect arguments concerning the isotropy of the primordial black body radiation. Finally we can conclude that the models obtained here will represent not only the early stages of evolution but also the present stage of the universe.

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