Article

Kaluza-Klein Inflationary Universe in General Relativity

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Abstract

Kaluza-Klein inflationary universe in general relativity has been studied. To obtain the deterministic model of the universe, it has been considered that the energy-momentum tensor of particles almost vanishes in the course of the expansion of the universe and thereby total energy-momentum tensor reduces to vacuum stress tensor. This assumption leads to (i) $a \approx e^{Ht}$, where a is scale factor and H is Hubble constant. (ii) the effective potential $V(\phi) = $ constant , where ϕ is Higg's field. It is observed that inflationary scenario is possible in Kaluza-Klein universe.

Keywords: Kaluza-Klein space-time, inflationary universe.

1. Introduction

Weinberg(1986) studied the unification of the fundamental forces with gravity which reveals that the space-time should be different from four. Since the concept of higher dimensional space-time is not unphysical, the string theories are discussed in ten dimensions or twenty six dimensions of space-time. Because of this, studies in *n*-dimensions inspired many researchers to enter into such field of study to explore the hidden knowledge about the universe. Chodos and Detweller(1980), Ibanez and Verdaguer (1986), Gleiser and Diaz(1988), Banerjee and Bhui(1990), Reddy and VenkateswaraRao (2001), Khadekar and Gaikwad(2001), Adhav *et al*.(2008) have studied the multidimensional cosmological models in Einstein's general relativity theory.

The theory of five dimensions is due to the idea of Kaluza (1921) and Klein(1926). A five dimensional [5D] general relativity is the best outcome of an attempt made by these two by using one extra dimension to unify gravity and electro-magnetism. Many researchers [Lee (1984), Appelquist *et al*. (1987), Collins *et al*. (1989), Overdin & Wesson (1997)] have used this concept for studying the models of cosmology and particle physics. According to Wesson (1984, 1999) and Bellini (2003) , the matter is induced in *4D* by *5D* vacuum theory for studying the cosmology of *5D* with pure geometry in non-compact Kaluza-Klein theory.

We are aware of the fact that the outstanding problems in cosmology like homogeneity, isotropy, horizon, flatness and primordial monopole problem in grand unified field theory are significantly solved by inflationary universes. The concept of early inflationary phase in grand unified field theories was introduced by Guth (1981) where symmetry breaking phase transition occurs with the decrease of temperature at

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the very early stages of evolution of universe [Zel'dovich *et al.*(1978); Kirzhnits *et al.*(1976,1977); Linde (1979)]. The life time of super cooled symmetric phase has been considered at $\phi = 0$, where ϕ is Higg's scalar field which breaks the symmetry. In this case, the energy-momentum tensor of particles almost vanishes in the course of the expansion of the universe and thereby total energy-momentum tensor reduces to vacuum stress tensor. This situation leads to (i) $a \approx e^{Ht}$, where a is scale factor and H is Hubble constant at that time which is given by

$$H = \left[\left(\frac{8\pi}{3M_p^2} \right) V(0) \right]^{\frac{1}{2}} \quad ,$$

where $M_p \approx 10^9 G_e V$ is the Planck mass [Tolman,1969].

Zel'dovich & Novikov (1975) proved that the symmetry breaking phase transition takes place at low temperature Tc where all vacuum energy $V(0)$ transforms into thermal energy and the universe is reheated up to the high temperature $T_1 \approx V(0)^{1/4}$ where further evolution starts. It has been proved by Rothman & Ellis (1986) that the problem of isotropy can be solved. Stein-Schabes (1987) has shown that the inflation will take place if the effective potential $V(\phi)$ has flat region where Higg's field ϕ evolves slowly but the universe expands in an exponential way due to vacuum field energy.

The significance of inflation for isotropization of the universe has been explained by Anninos *et al*. (1991). The inflationary scenario in the large scale structure of the universe has been studied by Panchapakeshan & Sethi (1992). Schmidt (1993) and Burd (1993) discussed inflationary scenario for FRW universe. Bali & Jain (2002) examined inflation in LRS Bianchi type-I space time in the presence of mass less scalar field with flat potential. Reddy *et al*. (2009) investigated inflationary scenario in Kantowski-Sache space time. Recently, Bali (2011) discussed inflationary scenario in Bianchi type-I space time by considering scale factor $a \approx e^{Ht}$ as used by Kirzhnits (1977) and Kirzhnits & Linde (1976).

With this motivation, Kaluza-Klein inflationary universe in general relativity has been studied.

2. Metric and Field Equations

We consider the Kaluza- Klein space-time in the form

$$ds^2 = -dt^2 + A^2 \left(dx^2 + dy^2 + dz^2 \right) + B^2 d\varphi^2 \quad , \qquad (2.1)$$

where A and B are functions of t only. The extra dimension is taken to be space-like.

Using Stein-Schabes (1987) approach, the Lagrangian of gravity minimally coupled with Higg's scalar field ϕ having effective potential $V(\phi)$ is given by

$$S = \int \left(\sqrt{-g}\right) \left[R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] d^4 x \tag{2.2}$$

The variation of this action S with respect to the dynamical field leads to the Einstein field equations (Here in geometrical units $8\pi G = 1 = c.$)

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \qquad , \tag{2.3}$$

where

$$T_{ij} = \partial_i \phi \partial_j \phi - \frac{1}{2} \left[\partial_p \phi \partial^p \phi + V(\phi) \right] g_{ij} \tag{2.4}$$

and

$$\frac{1}{\sqrt{-g}} \partial_i \left[\sqrt{(-g)} \partial_i \phi \right] = -\frac{dV}{d\phi} \tag{2.5}$$

Now, the Einstein field equations (2.3) for metric (2.1) with the help of equations (2.4) give a set of equations

$$3\left(\frac{\dot{A}}{A}\right)^2 + 3\frac{\dot{A}\dot{B}}{AB} = \frac{1}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + V(\phi) \, , \tag{2.6}$$

$$2\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \left(\frac{\dot{A}}{A}\right)^2 + 2\frac{\dot{A}\dot{B}}{AB} = -\frac{1}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + V(\phi) \qquad , \tag{2.7}$$

$$3\frac{\ddot{A}}{A} + 3\left(\frac{\dot{A}}{A}\right)^2 = -\frac{1}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + V(\phi) \, , \tag{2.8}$$

Equation (2.5) for scalar field ϕ leads to

$$\ddot{\phi} + \left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{\phi} + \frac{dV}{d\phi} = 0 \quad , \tag{2.9}$$

where dot $(\cdot)$ indicates the derivative with respect to t.

3. Solution of the Field Equations:

We assume that region is flat, so for flat region the effective potential is constant.

$$V(\phi) = \text{constant} = k \text{ (say)} \tag{3.1}$$

Solving equation (2.9) , we get

$$\dot{\phi} = \frac{l}{A^3 B} \tag{3.2}$$

where l is constant of integration.

To get a deterministic model, we consider the condition [Zel'dovich *et al*. (1978); Kirzhnits (1977)] that the scale factor can be expressed as

$$a = e^{Ht} \quad .$$

This leads to

$$a^4 = A^3 B = e^{4Ht} \tag{3.3}$$

Using equation (3.3) in equation (3.2), we have

$$\dot{\phi} = le^{-4Ht} \tag{3.4}$$

Subtracting equation (2.6) from equation (2.7), we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + 2\left(\frac{\dot{A}}{A}\right)^2 - 2\frac{\dot{A}\dot{B}}{AB} = 0$$

This gives

$$\left(\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B}\right) + \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) = 0$$

Integrating the above equation, we get

$$\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = \frac{L}{A^3 B} = \frac{L}{e^{4Ht}} \quad , \tag{3.5}$$

where L is constant of integration.

Using equations (3.3) and (3.5), we get

$$A = M e^{Ht} \exp\left(-\frac{L}{16H} e^{-4Ht}\right) \qquad (3.6)$$

$$B = N e^{Ht} \exp\left(\frac{3L}{16H} e^{-4Ht}\right) \qquad , \qquad (3.7)$$

where M and N are constants of integration.

Using equations (3.6) and (3.7) in equation (2.1) and after suitable transformation of co-ordinates with choice of constants, the Kaluza-Klein inflationary model is given by

$$ds^2 = -dt^2 + e^{2Ht} \exp\left[\frac{-L}{8H} e^{-4Ht}\right]\left(dX^2 + dY^2 + dZ^2\right) + e^{2Ht} \exp\left[\frac{3L}{8H} e^{-4Ht}\right] d\Psi^2$$

$$(3.8)$$

4. Physical Properties

Integrating equation (3.4), we get Higg's scalar field

$$\phi = s - \frac{l}{4H} e^{-4Ht} \qquad , \qquad (4.1)$$

where s is constant of integration.

The average expansion anisotropy parameter is defined as

$$\overline{A} = \frac{1}{4} \sum_{i=1}^{4} \left(\frac{\Delta H_i}{H}\right)^2 \qquad ,$$

where $\Delta H_i = H_i - H$

$$H_1 = H_2 = H_3 = \frac{\dot{A}}{A}, \quad H_4 = \frac{\dot{B}}{B}$$

$$\overline{A} = \frac{3}{16H^2} L^2 e^{-8Ht} \qquad (4.2)$$

The shear scalar σ^2 is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{4} H_i{}^2 - 4H^2\right) = 2AH^2$$

$$\sigma^2 = \frac{3}{8} L^2 e^{-8Ht} \qquad (4.3)$$

The deceleration parameter q is given by

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -1 \qquad\qquad (4.4)$$

5. Discussion and Conclusion

(i) The special volume is given by equation (3.3). It increases as time increases. Thus, inflationary scenario exists in Kaluza-Klein universe.

(ii) The equation (4.1) gives Higg's scalar field ϕ. It decrease slowly as time increases.

(iii) The average expansion anisotropy of the universe is given by equation (4.2).

For isotropy, we need $\overline{A} = 0$.

The equation (4.2) leads to $L = 0$.

Therefore, Kaluza-Klein universe isotropizes when $L = 0$.

(iv) This condition $L = 0$ implies that the shear scalar σ^2 given by (4.3) vanishes [is equal to zero] for isotropy.

(v) From equation (4.4), we get that the deceleration parameter is $q = -1$. Hence, Kaluza-Klein universe has exponential expansion or de-Sitter expansion. Thus, the model (3.8) approaches de-Sitter universe.

It has been shown that Kaluza-Klein metric (3.8) is isotropized under the special condition as pointed out by Rothman & Ellis (1986). Finally, one may conclude that the inflationary scenario is possible in Kaluza-Klein universe.

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