**Report**

# The Realization of Topological Qubits in Many-Sheeted Spacetime

Matti Pitkänen [1]

**Abstract**

Microsoft announced that it has created the first topological quantum computer utilizing topological qubits realized as condensed matter Majorana fermions. The condensed matter Majorana fermions are superpositions of fermions and holes: this breaks fermion number conservation or at least, the superselection rule for fermion number. The hole should correspond to a fermion "somewhere else". The many-sheeted space-time of TGD allows us to understand "somewhere else" as a second space-time sheet, a magnetic monopole flux tube. This leads to a model in which the Majorana Dirac equation is replaced with a description which respects fermion number conservation and super selection rule. TGD also predicts that the hierarchy of Planck constant makes topological superconductivity possible at physiological temperatures: biology would be the basic example.

TGD also leads to a generalization of the description in terms of Majorana fermions. It is based on the theoretical vision of TGD. The Galois group would serve as a generalization of the group $Z_2$ defining the parity of Majorana fermion. TGD predicts a 4-D variant of Galois group representing the transfers of fermions between different regions of the space-time surfaces identified in holography= holomorphy vision as roots $(f_1, f_2) = (0, 0)$ for an function pairs $H = M^4 \times CP_2 \to C^2$ analytic with respect to Hamilton-Jacobi coordinates generalizing complex coordinates. The Galois group is realized as analytic flows analogous to braidings mapping the roots to each other. The second Galois group is associated with dynamical complex analytic symmetries $g : C \to C$ $(f_1, f_2) \to (g \circ f_1, f_2)$. One can talk of number theoretic/topological n-ary digits for n-sheeted space-time surfaces. Pinary digits ($n$ is prime) are in a well-defined sense fundamental.

# 1 Introduction

Microsoft has unveiled Majorana 1 (thanks to Marko Manninen for sending the link), claimed to be the world's first quantum processor powered by topological qubits [1, 2] (see also the popular article at this).

## 1.1 How could one stabilize the computations and qubits?

The basic problem is how to realize computations in a stable way and how to make stable enough qubits? Concerning computation, topology comes to rescue.

1. Topological quantum computations (see this) can be represented as braidings, which are topologically stable under small deformations. Each braid strand represent a unitary evolution of a particle representing a unitary evolution if a qubit and the braiding operation would represent the computation. Braiding can be either time-like dynamical operation for point-like particles in plane or space-like for a braid connects two planes.

2. Since the 2-D plane containing particles as punctures, the homotopy group is non-abelian. This means that the rotation of a puncture around a second puncture of say bound state can transform the state such that transformation is not a mere phase factor but is a rotation which change the directions of the spins of the particles involved. Therefore the exchange of particles which can be seen as basic braiding operation changing the braid strands can induce an operation, which can be used as a basic building brick for a topological quantum computation.

[1] Correspondence: Matti Pitkänen http://tgdtheory.fi/. Address: Rinnekatu 2-4 A8, 03620, Karkkila, Finland. Email: matpitka6@gmail.com.

How could one obtain stable qubits? Qubit represented as a spin is not thermodynamically stable and extremely low temperatures are required. This is the case also for the proposed topological quantum computation: the reason is now that superconductivity is required and this is possible only at temperatures of order milli Kelvins. In any case, the notion of qubit should be topologized. How to achieve this? Here Majorana bound bound states have been proposed as an answer (see this).

1. Non-Abelian braid statistics, which means that their exchange realized as a 2-D rotational flow generated by braiding induces, instead of change of a sign in Fermi statistics, a non-Abelian unitary transformation of the state. It could be used to change the directions of their spins and affect the anyons.

   $2\pi$ rotation would induce a non-Abelian rotation instead of a mere sign change or phase factor in brain statistics. This is only possible in dimension 2 where the homotopy group can be non-abelian if there are punctures in the plane that the braids would represent. Similarly, swapping two Majorana fermions in braid produces a SU(2) rotation and can flip the spins and thus the qubits. This swap would be an essential operation in quantum computing. In order to have non-trivial topological quantum computation, one must have non-Abelian braid statistics characterized by a Lie group. Rotation group SO(2) or its covering SU(2) are the minimal options

2. The bound state of two Majorana fermions associated with planar punctures, anyons, would thus obey non-Abelian braid stastistics. It is also possible to affect the second fermion of Majorana bound state by rotating a puncture containing a fermion around the second fermion. Braidings could therefore represent unitary transformations having an interpretation as topological quantum computations.

   Wikipedia article mentions several realizations of Majorana bound states in superconductors. Quantum vortices in super conductors can provide this kind of states. The ends of the super-conducting wire or of line defects can contain the Majorana fermions. Also fractional Hall effect can provide this kind of states. The realization studied by Microsoft has the fermions of the Majorana fermion at the ends of a superconducting wire.

3. As I understand it, a condensed matter Majorana fermion would correspond formally to a superposition of an electron and a hole. The statistics would no longer be normal but non-Abelian Fermi statistic but would be that of a non-abelian anion.

   The weird sounding property of this statistics is that the the creation operator is equal to annihilation operator. One obtains two creation operators corresponding to two spin statse and square the creation operator of is unit operator: for fermions it vanishes. This implies that Majorana fermion number is defined only modulo 2 and only the number of fermions modulo 2 matters. Also the anticommutator of two creation operators at different points is equal to unit operator so that the system is highly nonlocal.

4. How the braiding could be realized? One can consider two options. Dance metaphor allows to understand the situation. Imagine that particles are dancers at the parquette. The dance would give rise to a time like braiding. If the feet of the dancers are tied to a wall of the dancing house by threads, also a space-like braiding is induced since the threads get tangled.

5. In the TGD framework, dancers would correspond to particle-like 3-surfaces moving in the plane and the dance would define the dancing pattern as a time-like braiding. This classical view is actually exact in the TGD framework since classical physics is an exact part of quantum physics in TGD. If thee particles are connected to the wall by threads realized as monopole flux tubes, a space-like braiding is induced.

6. These threads bring in mind the wires connecting superconductor and another object and containing Majorana fermions at its ends. Now the second end would be fixed and second would correspond

to a moving particle. Majorana bound states would correspond to the ends of the thread and the superconducting flow of the second end would correspond to the dynamical braiding.

## 1.2 Algebraic description of Majorana fermions

The dissertation of Aran Sivagure contains a nice description of Majorana fermions (see this). Majorana fermions would be quasiparticles possible in a many-fermion state. They would create from a fermion state with $N$ fermions a superposition of states with fermion numbers $N+1$ and $N-1$. They would be created by hermitian operators $\gamma_{n,1} = a_n^\dagger + a_n$ and $\gamma_{n,2} = i(a_n^\dagger - a_n)$ formed from the fermionic oscillator operators satisfying the standard anticommutation relations $\{a_m^\dagger, a_n\} = \delta_{m,n}$. Note that one consider also more general Hermitian operators $\gamma_{n,1} = exp(i\phi)a_n^\dagger + exp(-i\phi)a_n$ and $\gamma_{n,2} = i(exp(i\phi)a_n^\dagger - exp(-i\phi)a_n)$.

One can also form analogs of plane waves as superpositions of these operators $\gamma_{k,1} = \sum_n [exp(ikx_n)a_n^\dagger + exp(-ikx_n)a_n]/\sqrt{N}$ and $\gamma_{k,2} = i\sum_n [exp(ikx_n)a_n^\dagger - exp(-ikx_n)a_n]\sqrt{N}$. Here $N$ is the number of lattice points and discrete Fourier analysis is used.

The anticommutations would be $\{\gamma_{i,k_1}, \gamma_{i,k_2}\} = 2 \times Id\delta_{k_1,k_2}$, $i = 1,2$ where $Id$ denotes the unit operator. For different points $i \neq j$ the anticommutativity implies that the anticommutators vanish. Therefore the statistics are not the ordinary Bose- or Fermi statistics and non-Abelian statistics. The anticommutation relations reflect the fact that the application of the creation operators twice does not change the physical states so that the number of Majorana fermions is determined only modulo 2.

## 1.3 TGD description of the situation

The condensed matter Majorana fermions are superpositions of electrons and holes: this breaks fermion number conservation or at least, the superselection rule for fermion number. The hole should correspond to a fermion "somewhere else". In condensed matter, "elsewhere" could correspond to a conduction band in momentum space. The many-sheeted space-time of TGD allows us to understand "somewhere else" as a second space-time sheet, a magnetic monopole flux tube. This leads to a model in which the Majorana Dirac equation is replaced with a description which respects fermion number conservation and super selection rule. TGD also predicts that the hierarchy of Planck constant makes topological superconductivity possible at physiological temperatures: biology would be the basic example.

TGD leads to a generalization of the description in terms of Majorana fermions based on the number theoretical vision of TGD [4, 5]. The Galois group would serve as a generalization of the group $Z_2$ defining the parity of Majorana fermion. Two Galois groups are possible: the internal and external Galois group.

1. TGD predicts a 4-D variant of Galois group, the internal Galois group, representing the transfers of fermions between different regions of the space-time surfaces identified in holography= holomorphy vision as roots $(f_1, f_2) = (0,0)$ for function pairs $H = M^4 \times CP_2 \to C^2$ analytic with respect to Hamilton-Jacobi coordinates generalizing complex coordinates. The internal Galois group is realized as analytic flows analogous to braidings mapping the roots $(f_1, f_2) = (0,0)$ to each other and having as interfaces the regions at which two or more roots co-incide.

2. The simpler version of the external Galois group, is associated with dynamical complex analytic symmetries $g : C \to C$: $(f_1, f_2) \to (g \circ (f_1, f_2)$. In this case, the Galois group relates to each other disjoint space-time surfaces. When $g$ reduces to a map $g = (g_1, Id)C \to C$, where $g_1$ has no parametric dependence on $f_2$, one can assign to it an ordinary Galois group relating to each other the disjoint roots of $g_1 \circ f_1$, which are algebraic numbers.

   The notion of external Galois group generalizes. For the general case $g = (g_1, g_2)$, the roots of $g \circ f$ are disjoint space-time surfaces representing pairs of algebraic numbers $(f_1, f_2) = (r_{i,1}, r_{i,2})$. It is possible to assign to the roots the analog of the Galois group. This group should act as a group of automorphisms of some algebraic structure. This structure cannot be a field but algebra structure is enough. The arithmetic operations would be component-wise sum $(a,b) + (c+d) = (a+c, b+d)$

and componentwise multiplication $(a, b) * (c, d) = (ac, bd)$. The basic algebra would correspond to the points of $(x, y) \in E^2$ or rationals and the extension would be generated by the pairs $(f_1, f_2) = (r_{i,1}, r_{i,2})$. This structure has an automorphism group and would serve as a Galois group. The dimension of the extension of $E^2$ could define the value of the effective Planck constant.

3. In [5] the idea that space-time surfaces can be regarded as numbers was discussed. For a given $g$, one can indeed construct polynomials having any for algebraic numbers in the extension $F$ of $E$ defined by $g$. $g$ itself can be represented in terms of its $n$ roots $r_i = (r_{i,1}, r_{i,2})$, $i = 1, n$ represented as space-time surfaces as a product $\prod_i (f_1 - r_{i,1}, f_2 - r_{i,2})$ of pairs of monomials. One can generalize this construction by replacing the pairs $(r_{i,1}, r_{i,2})$ with any pair of algebraic numbers in $F$. Therefore all algebraic numbers in $F$ can be represented as space-time surfaces. Also the sets formed by numbers in $F$ can be represented as unions of the corresponding space-time surfaces.

# 2 Could many-sheeted spacetime allow a more fundamental description of Majorana like states?

The problematic aspect of the notion of Majorana fermion as a fundamental particle is that the many-fermion states in this kind of situation do not in general have a well-defined fermion number. Physically, fermion number conservation is a superselection rule so that the superposition of fermion and hole must physically correspond to a superposition of fermion states, where the hole corresponds to a fermion which is outside the system. Condensed matter Majoranas avoid this problem but the assumption of ill-defined fermion number seems phenological: holes must correspond to fermions which are somewhere else.

## 2.1 Could Majorana fermions corresponds excitations for which fermions are transferred between different space-time sheets

In TGD, the notion of many-sheeted space-time however suggests an elegant solution to the problem at the fundamental level and also suggests that the analogs of Majorana fermions and the associated superconductivity are possible at room temperatures.

1. In condensed matter physics Majorana fermions could be assigned with the vortices of superconductors. In the TGD Universe, these vortices could correspond to monopole flux tubes as body parts of the field body. The states created by $\gamma_i$ would be superpositions of states in which the fermion is at the monopole flux tube or at the normal space-time sheet representing the part of the condensed matter system that we see. The Majorana description would be an effective description.

2. The Majorana creation operators $\gamma_i$ would be replaced with operators which shift the fermion from ordinary space-time sheet to the monopole flux tube and vice versa. From the geometric interpretation it is clear that this operation must be idempotent. This operation must be representable in terms of annihilation and creation operators. The operators $\gamma_i$ would be expressible products of creation and annihilation operators acting at the space-time sheets 1 and 2.

   One can consider either commutation or anticommutation relations for these operators. Since the operation does not change the total fermion number, the interpretations as a bosonic operator can be argued to be natural so that commutation relations look more plausible.

3. Neglecting for a moment the indices labelling positions and spins and denoting the oscillator operators associated with the two space-time sheets $a$ and $b$ a rather general expression for the hermitian operators $\gamma_1$ and $\gamma_2$ would be

$$\gamma_1 = b^\dagger a + a^\dagger b \ \ , \quad \gamma_2 = i(b^\dagger a - a^\dagger b) \ \ .$$

Suppose fermionic anticommutations are satisfied. Only cross terms contribute to anticommutators (and also commutators).

4. Anticommutators are given by

$$2\gamma_1^2 = 2\gamma_2^2 = b^\dagger a a^\dagger b + a^\dagger b b^\dagger a = a^\dagger a - b^\dagger b = N(a) + N(b) - 2N(a)N(b) \ .$$

$$\{\gamma_1, \gamma_2\} = 0 \ .$$

The eigenvalues of $N(a) + N(b) - 2N(a)N(b)$ vanish for $(N_a, N_b) \in \{(1,1),(0,0)\}$ and are equal to 1 for $(N_a, N_b) \in \{(1,0),(0,1)\}$. The result implies that the squares of the operators $\gamma_i$ act like an identity operator, which conforms with the Majorana property. The two operators would anticommute.

5. One can also consider the commutator, which could be argued to be more natural on the basis of the physical interpretation as a hermitian observables. In this case one has trivially $[\gamma_i, gamma_i] = 0$ and $[\gamma_1, gamma_2] = N(a) - N(b)$. The commutator would vanish only for $N(a) = N(b)$ and the physical states could be eigenstates of only $\gamma_1$ or $\gamma_2$ as an observable. In any case, the Majorana-like property would hold true.

One can also form analogs of plane waves as superpositions of these operators

$$\gamma_{k,1} = \sum_n [exp(ikx_n) b_n^\dagger a_n + exp(-ikx_n) a_n^\dagger b_n]/\sqrt{N} \ ,$$

$$\gamma_{k,2} = i\sum_n [exp(ikx_n) b_n^\dagger a_n - exp(-ikx_n) a_n^\dagger b_n]/\sqrt{N} \ .$$

Here $N$ is the number of lattice points and discrete Fourier analysis is used. The commutators and anticommutators vanish for different points. Assume that the occupations numbers $N(a,n)$ and $N(b,n)$ do not depend on $n$ so that one $N(a,n) = N(a)$ and $N(b,n) = N(b)$.

1. The anticommutators are given

$$\{\gamma_{k_1,1}, \gamma_{k_2,1}\} = \{\gamma_{k_1,2}, \gamma_{k_2,2}\} = (N(a) + N(b) - 2N(a)N(b))\delta_{k_1+k_2}/N \ .$$

$$\{\gamma_{k_1,1}, \gamma_{k_2,2}\} = 0 \ .$$

The analog of the Majorana property is true and reflects the fact the transfer operator is classically idempotent.

2. The non-trivial commutators are

$$\{\gamma_{k_1,1}, \gamma_{k_2,2}\} == (N(a) - N(b))\delta_{k_1,k_2}/N \ .$$

$\gamma_{k_1,1}$ and $\gamma_{k_2,2}$ can be regarded as non-commuting observables.

## 2.2 $OH - OH^- + p$ as topological qubit?

While writing this, I noticed that the $OH - OH^- + p$ qubits, where p is a dark proton ag monopole flux tubes, that I proposed earlier to play fundamental role in biology and perhaps even make quantum counterparts of ordinary computes possible [6], are to some degree analogous to Majorana fermions. The extremely nice feature of these qubits would be that superconductivity, in particular bio-superconductivity, would be possible at room temperature. This is would be possible by the new physics predicted by TGD both at the space-time level and at the level of quantum theory.

1. In TGD space-times are surfaces in $H = M^4 \times CP_2$ and many-sheetedness is the basic prediction. Another related prediction is the notion of field body (magnetic/electric) body. Number theoretica view of TGD predicts a hierarchy of effective Planck constants making possible quantum coherence in arbitrarily long length scales. Second new element is zero energy ontology modifyin profoundly quantum measurement theory and solving its basic problem.

2. $OH - OH^- + p$ qubit qubit means that one considers protons but also electrons can be considered. Now the proton is either in the OH group associated with water molecule in the simplest situation in which Pollack effect occurs or the proton is a dark proton at a monopole flux tube. A proton in OH would be analog of non-hole state and the dark proton in the flux tube be the anaog of hole state.

3. What is new is that the proton being on/off the spacetime surface would represent a bit. For Majorana fermions, the situation is rather similar: the hole state corresponds to the electron being "somewhere else", which could also correspond to being on a monopole flux tube as I have suggested. In standard quantum computation, a qubit would correspond to a spin.

4. If the energies for OH and $OH - OH^- + p$ bits are close to each other, the situation is quantum critical and the qubits can be flipped and a process similar to quantum computation becomes possible. Also superconductivity becomes possible at the magnetic flux tubes analogous to magnetic vortices appearing in superconductivity and in fractional Quantum Hall effect.

   These are truly topological qubits also because the topologies of the spacetime surface for different bit values are different. However, the energy difference must be larger than the thermal energy, otherwise the qubits become unstable. With the help of electric fields, qubits can be sensitized to quantum criticality and their inversion becomes possible.

5. The above argument suggests that a non-abelian statistics could be understood for $OH - OH^- + p$ qubits. The anticommutation/commutation relations for the operators trasferring protons to the magnetic body would not be identical to those for Majorana oscillator operators the squares of these operators would be proportional to unit operator which is essentially the Majorana property.

   I have proposed a possible realization for this in a more general case. The exchange of dark protons/qubits would be induced by reconnection of monopole flux tubes: it would therefore be a purely topological process. Nothing would be done to the dark electrons, but the flux tubes would be reconnected. Strands AB and $CD$ would become strands AD and BC. At the same time, the unilluminated protons would become associated with different $O^-$. In this exchange, could the final result be represented as an SU(2) rotation for the entire space.

6. The transfer of proton from OH to magnetic monopole flux tube would correspond to the Majorana like quasiparticle. In zero energy ontology (ZEO) [3], point-like particles are replaced with 3-surfaces and holography forces to replace them with their 4-D Bohr orbits. The Majorana quasiparticle would classically correspond to a Bohr orbit leading from proton in H to dark proton at the monopole flux tube.

Received March 31, 2025; Accepted January 3, 2026

# References

[1] David Aasen et al. Roadmap to fault tolerant quantum computation using topological qubit arrays, 2025. Available at: https://arxiv.org/abs/2502.12252.

[2] Microsoft Azure Quantum et al. Interferometric single-shot parity measurement in InAsAl hybrid devices, 2025. Available at: https://www.nature.com/articles/s41586-024-08445-2.

[3] Pitkänen M. Some comments related to Zero Energy Ontology (ZEO). Available at: https://tgdtheory.fi/public_html/articles/zeoquestions.pdf., 2019.

[4] Pitkänen M. A fresh look at $M^8 - H$ duality and Poincare invariance. https://tgdtheory.fi/public_html/articles/TGDcritics.pdf., 2024.

[5] Pitkänen M. About Langlands correspondence in the TGD framework. https://tgdtheory.fi/public_html/articles/Frenkel.pdf., 2024.

[6] Pitkänen M. Quartz crystals as a life form and ordinary computers as an interface between quartz life and ordinary life? https://tgdtheory.fi/public_html/articles/QCs.pdf., 2024.

[7] Pitkänen M. The realization of topological qubits in many-sheeted space-time. https://tgdtheory.fi/public_html/articles/majorana.pdf., 2025.