Article

On Complex Dynamics & Primordial Structure Formation

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Abstract

Conventional wisdom says that the formation of large-scale structures in cosmology follows from the evolution of density perturbations under the combined effect of gravitation and cosmic expansion. We survey here the reasons why this framework fails to hold in primordial cosmology, due to the severe limitations placed on the Friedmann-Robertson-Walker (FRW) metric, the continuity hypothesis and the standard equations of fluid flows. We insist that understanding primordial cosmology must rely instead on a complex dynamics model of *evolving dimensional fluctuations,* conjectured to come into play far above the electroweak scale. A key outcome of this model is the universal generation of *topological defects and condensates* emerging from the complex Ginzburg-Landau equation.

Keywords: Scalar density perturbation, primordial universe, FRW cosmology, structure formation, complex dynamics, dimensional fluctuations.

1. Introduction

Relativistic cosmology models the Universe as a nearly homogeneous and isotropic expanding spacetime, as described by the FRW metric. In this framework, the observed large-scale structure of the Universe (galaxies, clusters, cosmic web including voids, filaments, halos etc.) develops from small perturbations of the smooth overall background. Perturbations are studied through *cosmological perturbation theory (CPT)*, which tracks how primordial inhomogeneities evolve into large-scale structures from fluctuations in energy density, pressure, and spacetime curvature. Despite its undisputed successes, CPT remains however inadequate for the study of early Universe formation, on spacetime-scales approaching the Planck regime.

The goal of this report is to survey the reasons why CPT fails to hold in primordial cosmology, due to the severe limitations placed on the FRW metric, the continuity hypothesis and the standard equations of fluid flows. In our view, understanding of primordial cosmology must rely instead on a complex dynamics model of *evolving dimensional fluctuations,* conjectured to come

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into play far above the electroweak scale. A key outcome of this model is the universal generation of *topological defects and topological condensates* emerging from the complex Ginzburg-Landau equation (CGLE).

The report is organized as follows: next section is a condensed review of how scalar density perturbations are dealt with in standard cosmology. Section 3 goes over the limitations of the FRW model and CPT in explaining structure formation in primordial cosmology. Section 4 details our perspective on the *complex dynamics* of primordial dimensional fluctuations and their impact on structure formation via CGLE and self-organization.

For the sake of clarity and accessibility, the report is designed in a "user-friendly" format, with emphasis on pedagogical exposition rather than formal derivations.

2. Scalar Density Perturbations in Standard Cosmology

We start by reviewing the evolution equation of scalar density perturbations in the standard model of cosmology (the Lambda-CDM model) [1 - 4]. Let $\rho(\vec{x},t)$ denote the local matter or energy density. The main parameter of interest in CPT is the *density perturbation*, which quantifies the local density deviation normalized to the average density of the Universe,

$$\delta(\vec{x},t)=\frac{\Delta\rho(\vec{x},t)}{\overline{\rho}}=\frac{\rho(\vec{x},t)-\overline{\rho}}{\overline{\rho}} \tag{1}$$

The density perturbation (1) can be studied in Fourier space using the decomposition in modes of momenta $\vec{k}$,

$$\delta(\vec{x},t)=\int d^3k\,\delta_k(t)\exp(i\vec{k}\cdot\vec{x}) \tag{2}$$

Statistical distribution of modes is described using the concept of *power spectrum* $P(k)$. Observations indicate that the amplitude of density perturbations is *nearly scale invariant*, meaning that it stays roughly the same across all length scales. As a result, primordial fluctuations carry about the same strength and large- and small-scale structures start off with nearly similar density perturbations. In terms of the power spectrum, one has

$$P(k)\propto k^{n_s-1} \tag{3}$$

in which n_s denotes the *scalar spectral index*. While $n_s = 1$ corresponds to exact scale invariance, observations hint that $n_s \approx 0.965$, which shows an actual slight red tilt (meaning more power allocated to large scales).

The linear scalar perturbations to a flat FRW metric takes the form

$$ds^2 = a^2(\eta)[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j] \tag{4}$$

where η is the conformal time and Ψ, Φ are scalar potentials (with $\Psi = \Phi$, if there are no anisotropic stresses). Relation (2) defines the FRW metric in the so-called *conformal Newtonian gauge*. There are two key equations of CPT in Fourier space, namely,

a) The *continuity equation* reflecting energy conservation and given by,

$$\dot{\delta} + (1+w)(\theta - 3\dot{\Phi}) + 3H(c_s^2 - w)\delta = 0 \tag{5}$$

b) *Euler equation* reflecting momentum conservation and written as,

$$\dot{\theta} + H(1-3w)\theta + \frac{\dot{w}}{1+w}\theta - \frac{k^2 c_s^2}{1+w}\delta - k^2\Psi = 0 \tag{6}$$

Here, $H = a'/a$ is the conformal Hubble parameter, $a' = da/d\eta$, $w = \overline{p}/\overline{\rho}$, $c_s^2 = \delta p/\delta\rho$ is the square of the sound speed and $\theta = i k^j v_j = i\vec{k}\cdot\vec{v}$ stands for the velocity potential with wavevector $\vec{k}$.

Using (5) – (6), the equation for the evolution of scalar perturbations in the Lambda-CDM Universe assumes the form,

$$\boxed{\ddot{\delta} + 2H\dot{\delta} - 4\pi G\overline{\rho}\,\delta = 0} \quad \text{(Newtonian limit)} \tag{7}$$

The full relativistic equation beyond the Newtonian limit (7) reads

$$\boxed{\ddot{\delta} + A(H,w)\dot{\delta} + B(c_s^2, k^2, a^2, w, \overline{\rho})\delta = S(\Phi)} \tag{8}$$

in which $A(...)$, $B(...)$ are functions of variables indicated in (8) and $S(\Phi)$ is the source term induced by Φ.

3. Limitations of FRW Cosmology in the Primordial Universe

While being adequate in describing the late dynamics of the Universe, FRW cosmology faces crucial limitations when applied to the evolution of the primordial Universe:

1) Along with General Relativity (GR), FRW breaks down near the Big Bang singularity where large and highly unstable curvature fluctuations ruin the *smooth topology of ordinary four-dimensional spacetime*. It is also conceivable that, in line with the Dimensional Reduction conjecture [11], spacetime dimensionality drops down in some continuous but unpredictable fashion.

2) The cosmological principle posits a smooth (differentiable) and uniform Universe, yet these assumptions are bound to be violated near the Big-Bang singularity, as hinted by the Sakharov conditions for baryogenesis and the far-from-equilibrium properties of Planck physics. In the same context, strong nonlinearities associated with (1), that is, $\delta = O(1)$ are prone to ruin the basis of the entire CPT model.

3) FRW does not naturally explain the horizon and flatness puzzles of cosmology without fine-tuned initial conditions—issues typically resolved by *inflation*, which is still hypothetical and external to the Lambda-CDM paradigm.

4) FRW lacks a mechanism of structure formation from primordial fluctuations as it serves only as a background for inflationary cosmology.

5) The relevance of quantum gravity effects near the Planck scale remains highly controversial [14].

6) The continuity equation (5) fails when mass/energy/momenta are not conserved due to the presence of sources or sinks. Likewise, Bianchi identities of GR break down under strong curvature fluctuations (the local spacetime manifold becomes non-smooth and singular) or when the effects of deterministic or stochastic chaos *cannot be discarded* (as implied from the generic nonintegrability of Hamiltonian systems [15]).

7) Fluid equations of the type (6) become invalid in the presence of boundary layers, unpredictable dissipation or turbulence, shocks, non-Newtonian stress-strain relationship or rarefied flow conditions.

5. Beyond CPT with Complex Dynamics of Dimensional Fluctuations

It is generally believed that, if the ongoing cosmological observations continue to diverge from predictions, our current formulation of Lambda-CDM may require a fundamental paradigm shift. Parallel with the current situation in cosmology, independent research indicates that nonlinear dynamics of far-from-equilibrium systems has *universal features* linking the complex Ginzburg-Landau equation (CGLE) with the tenets of Quantum Field Theory (QFT), Standard Model of Particle Physics (SM) and the high-energy regime of primordial gravitation [16 – 18]. Here we consolidate this view by arguing that CGLE lies behind structure formation in cosmology and the emergence of *topological defects and condensates* in high-energy physics. From this standpoint, *complex dynamics* appears to provide the most sensible way out of the current crisis in cosmology. Building upon the ideas of [12], the next paragraph elaborates on the emergence of CGLE from the dynamics of dimensional fluctuations.

5.1 CGLE from Dimensional Fluctuations above the Electroweak Scale

Reaction-Diffusion (RD) processes are a subset of complex phenomena defined within the framework of Nonequilibrium Statistical Physics. These models are typically formulated in $d+1$ dimensions, where d is the dimension of the Euclidean manifold representing the physical space and t is the time coordinate. Ref. [12] develops a toy RD model acting on a two-dimensional lattice $(d=2)$, whose local variables are time-varying *dimensional fluctuations* $\delta\varepsilon(t)=\delta[2-d(t)]$. The model includes a *scattering* event at rate D, a *clustering* event at rate u and a *decay* (or *percolation*) event at rate $\kappa=\lambda-\lambda_c$, with λ being a control parameter nearing its critical value λ_c. Up to a leading order approximation, the macroscopic properties of RD processes may be encoded in a *mean-field* (MF) equation, which quantifies the competition between losses and gains in a generic density parameter $\rho(t)$. In particular, the decay/percolation process occurs with a rate proportional to $\kappa\rho(t)$ and leads to a gain in density. By contrast, the clustering process drops the density with a rate proportional to $u\rho^2(t)$. Ignoring diffusion, the resulting MF equation takes the form

$$\frac{\partial\rho(t)}{\partial t}=\kappa\rho(t)-u\rho^2(t) \tag{9}$$

In the context of [12] the control parameter $\lambda(t)=\lambda[\delta\varepsilon(t)]$ represents the *density of dimensional fluctuations* $\delta\varepsilon(t)<<1$ while $\rho(t)$ denotes the *density of active (or unstable) lattice sites.* A straightforward extrapolation of (9) is given by the system of coupled partial differential equations

$$\frac{\partial \rho_1(x,t)}{\partial t} = D_1 \Delta \rho_1(x,t) + f(\rho_1, \rho_2, \kappa) \tag{10a}$$

$$\frac{\partial \rho_2(x,t)}{\partial t} = D_2 \Delta \rho_2(x,t) + g(\rho_1, \rho_2, \kappa) \tag{10b}$$

According to [12] and references therein, an arbitrary solution of (10) lying near the bifurcation point at $\kappa > \kappa_0$ can be expressed through a *complex-valued function* $\psi(r,\tau)$ obeying the CGLE in one spatial dimension,

$$\boxed{\frac{\partial \psi}{\partial \tau} = \psi + (1+i\alpha)\frac{\partial^2 \psi}{\partial r^2} - (1+i\beta)\psi|\psi|^2} \tag{11}$$

Here, the set of new coordinates is given by

$$r = \sigma x \tag{12a}$$

$$\tau = \sigma^2 t \tag{12b}$$

where,

$$\sigma = (\kappa - \kappa_0)^{1/2} \propto (\lambda - \lambda_c)^{1/2} << 1 \tag{13}$$

As it is known, CGLE represents a nonlinear partial differential equation which serves as a universal model for the onset of spatiotemporal dynamics in nonlinear systems. It is especially helpful in the study of critical behavior where order parameters emerge – such as superfluidity, superconductivity, turbulence, Bose-Einstein condensation, nonlinear waves and pattern formation. In (11) $\psi(r,\tau)$ acts as order parameter, α, β are real coefficients accounting for diffusion and nonlinear effects, respectively. Stated differently, the term $\psi(r,\tau)$ stands for the linear growth or decay, the Laplacian measures the contribution of diffusion and the nonlinear term $\psi|\psi|^2$ embodies the contribution of self-interaction.

5.2 Cosmological Structure Formation from CGLE

Pattern/structure formation and self-organization emerging from CGLE is a rich and active research topic, with applications across various disciplines. To avoid excessive information and to stimulate further independent analysis, here we limit the discussion to few representative examples.

a) **Topological defects** are localized regions where the order parameter is *singular* or undefined – like points, lines or surfaces around which the phase of ψ winds in some nontrivial manner. In two- and three dimensions, typical examples include *vortices* (points where $|\psi| = 0$ and the phase of $\psi = \arg(\psi)$ winds by $2\pi n$ around the core), *domain walls* are interfaces between regions with different phase or amplitude configurations, and *dislocations/spirals* which are phase defects of wavefronts in pattern forming systems.

In the context of CGLE, topological defects emerge from *broken symmetry* and solutions with *nontrivial phase structures*. They form spontaneously from instabilities, noise or as remnants of quenched phase transitions (Kibble-Zurek mechanism). The so-called winding number (or *topological charge*) quantifies the phase change around defects.

b) **Topological condensates** represent coherent and long-lived, often self-organized macroscopic structures formed by the aggregation of topological defects.

Fig. 1 below is a visualization of a topological vortex arising from CGLE. The left panel shows the amplitude of the order parameter $|\psi|$, which vanishes at the center (the vortex core). The right panel shows the phase, which winds by 2π around the core and defines the topological charge of the vortex.

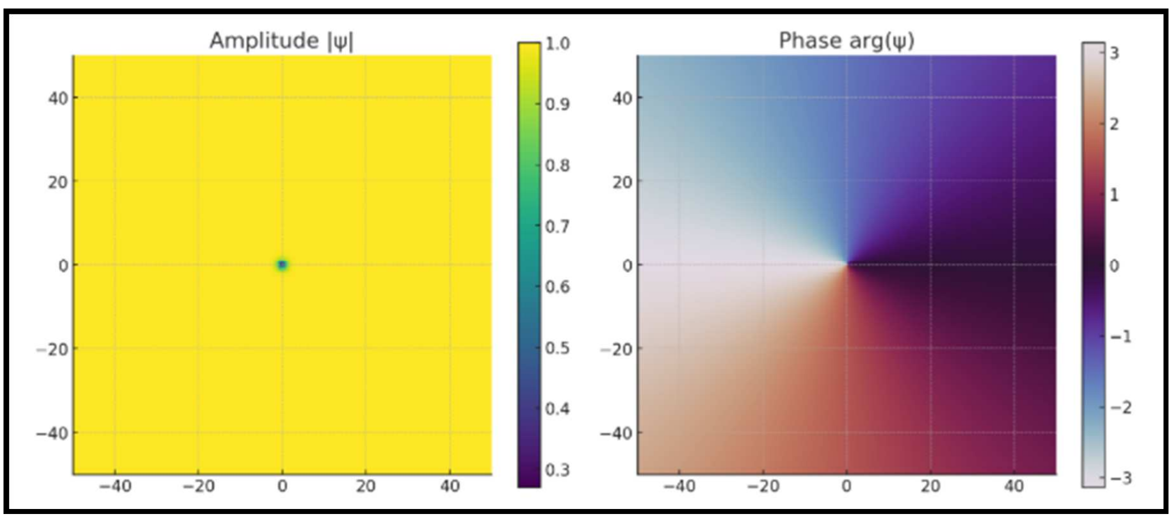
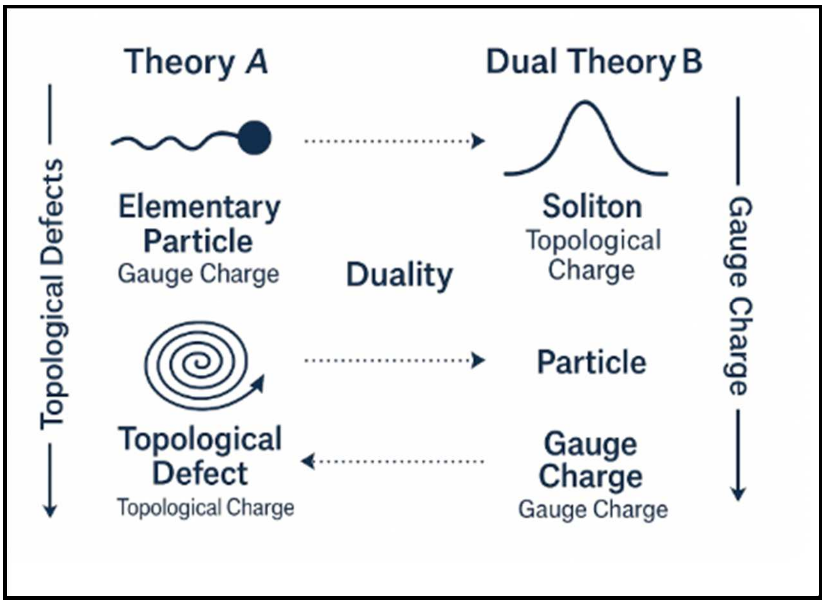


Fig. 1. Visualization of a topological vortex derived from CGLE.

It is instructive to note that topological defects can be regarded as *dual objects* to the elementary particles of high-energy physics, as highlighted by the table and symbolic diagram shown below. According to this view, there is an intriguing analogy in how symmetries and topology lead to conserved quantities and defects [5 – 10].

Theory A	Dual Theory B
Particles with gauge charges	Topological defects with winding numbers
Electric field lines	Magnetic monopole worldlines
Quarks/gluons in QCD	Confining flux tubes or dual superconductor
Elementary bosons	Vortices or solitons in dual theory
Gauge theory with local symmetry	Topological theory with global symmetry



Vortex lattices derived from CGLE are periodic arrangements of vortices, forming structured patterns in the complex amplitude field. They represent a self-organized state of the underlying nonlinear system, often found in 2D simulations or real-world systems near oscillatory instabilities. The evolution of vortex lattices is a complex interplay of self-organization, turbulence, and external influences, leading to a variety of dynamic phases and structural transitions. Fig 2 displays snapshots of a two-dimensional order parameter $|\psi(x,y,t)|^2$ taken at various time intervals. It illustrates how structure changes dynamically over time, exhibiting features common in the evolution of vortex lattices and cosmic voids / halos.

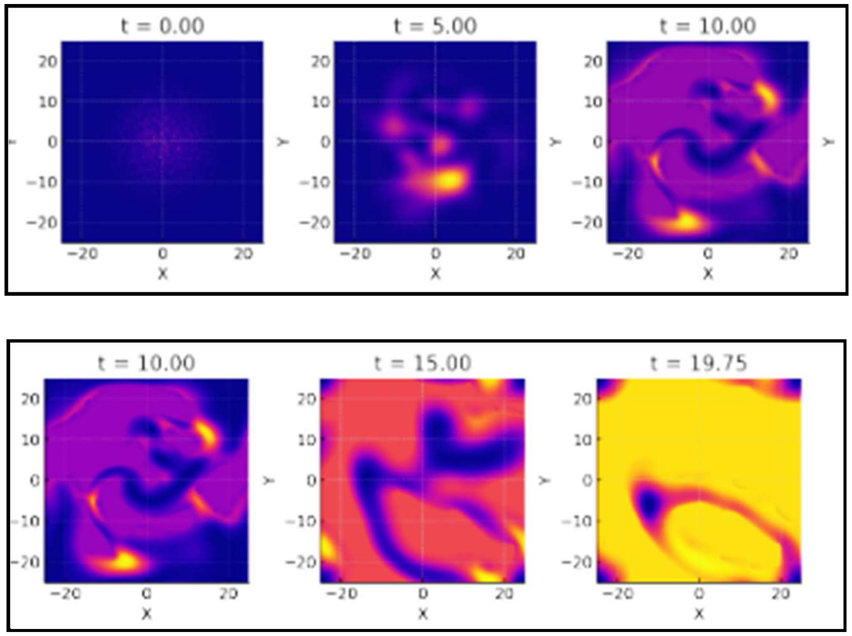


Fig. 2. Evolving 2D vortex lattices as analog of cosmic voids.

Fig. 3 visualizes the evolution of vortex lines in the three-dimensional (3D) CGLE, which echoes the formation of helical strings in the cosmic web. This structure develops from the instability of vortex filaments in the high-dispersion limit of (11), namely $\alpha >> 1$ [13].

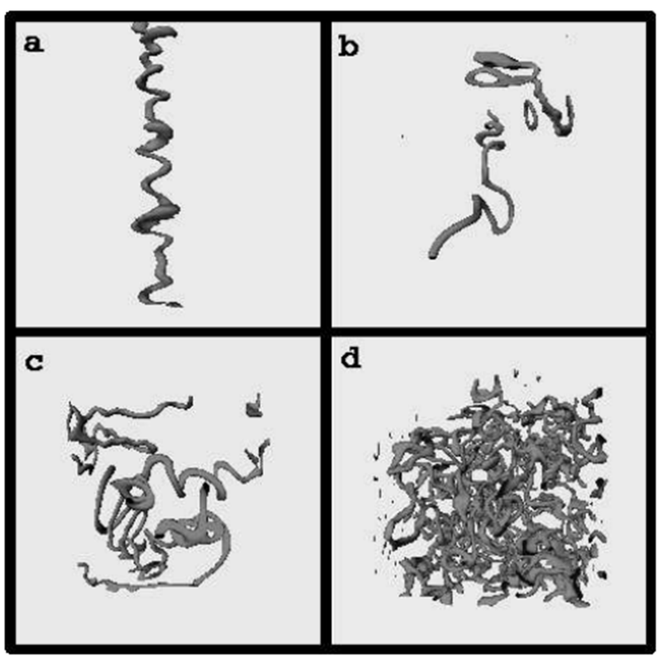


Fig. 3. Evolution of vortex lines in 3D CGLE.

6. Discussion

There are well motivated reasons why, due to their inherent assumptions and constraints, both FRW and CPT models are inadequate for understanding structure formation in primordial cosmology. By contrast, there are compelling hints that a framework based on *complex dynamics of evolving dimensional fluctuations* is essential for explaining primordial cosmology, as it connects to phenomena occurring far above the electroweak scale. The proposed framework builds upon the universal generation of topological defects and condensates associated with the complex Ginzburg-Landau equation (CGLE). The main benefit of this proposal is that the premises of CGLE are *independent* from any formulation of classical gravity, classical Thermodynamics or the theory of fluid flows. In addition, the duality of cosmological structure formation and topological solutions of Quantum Field Theory is likely to bring unforeseen insights into future model building efforts.

Received April 26, 2025; Accepted December 10, 2025

References

1. Peebles, P.J.E., The Large-Scale Structure of the Universe (1980).
2. Dodelson, S., Modern Cosmology (2003).
3. Bernardeau, F., *et al*., "Large-Scale Structure of the Universe and Cosmological Perturbation Theory" (Physics Reports, 2002).
4. Ma, C.-P. & Bertschinger, E., "Cosmological Perturbation Theory in the Synchronous and Conformal Newtonian Gauges" (Astrophysical Journal, 1995).
5. Aranson, I.S. & Kramer, L., "The World of the Complex Ginzburg-Landau Equation" (Reviews of Modern Physics, 2002).
6. David Tong, Lectures on Quantum Field Theory, https://www.damtp.cam.ac.uk/user/tong/qft.html
7. Michael E. Peskin, Duality in Supersymmetric Yang-Mills Theory, preprint https://arxiv.org/abs/hep-th/9702094
8. Seiberg & Witten, Electric-Magnetic Duality, Monopole Condensation, and Confinement in N=2 Supersymmetric Yang-Mills Theory, Nucl. Phys. B426 (1994) 19–52.
9. Montonen & Olive, Magnetic Monopoles as Gauge Particles? Phys. Lett. B72 (1977) 117–120.
10. Franz, Tesanovic, Duality and Quantum Criticality in 2+1 Dimensional Superconductors, Phys. Rev. Lett. 87, 257003 (2001)
11. Goldfain, E., (2023) Dimensional Reduction as Source of Cosmological Anomalies, preprint http://dx.doi.org/10.32388/RX6BTE.
12. Goldfain, E., (2024), From Complex Dynamics to Foundational Physics, preprint http://dx.doi.org/10.32388/HC9S30

13. Aranson I. S. *et al*., (1997), The dynamics of vortex lines in the three-dimensional complex Ginzburg-Landau equation: instability, stretching, entanglement, and helices, preprint https://arxiv.org/abs/patt-sol/9712004

14. Goldfain, E., Open Limitations of Quantum Gravity: a Brief Overview, (2024), preprint http://dx.doi.org/10.13140/RG.2.2.36070.43841/3

15. Goldfain, E., Hamiltonian Chaos and Fractal Topology of Spacetime, (2023), preprint http://dx.doi.org/10.32388/7ZYVDB

16. Goldfain, E. Complex Dynamics and the Schrödinger Equation, (2024), preprint http://dx.doi.org/10.32388/AFBU2E.2

17. Goldfain, E., Complex Dynamics and Primordial Gravity, (2024), preprint http://dx.doi.org/10.13140/RG.2.2.20849.60007/1

18. Goldfain, E., On the Bifurcation Structure of Particle Physics, (2024), preprint http://dx.doi.org/10.13140/RG.2.2.14675.39209/1