Article

Bianchi Type-VI_0 Cosmological Model with Cloud String in *f*(*T*) Gravity

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Abstract

In this paper, we have investigated the cosmological models in the framework of *f*(*T*) modified theory of gravity. Investigated models have been deduced in the presence of cosmic string in the universe. The function *f*(*T*) such as $f(T)=\eta(-T)^{\alpha}$ is used for investigation. The physical and kinematical properties of the deduced models have been obtained and analyzed. The resemblance of the resultant models has been compared with the present day universe.

Keywords: *f*(*T*) gravity, Bianchi type-VI_0, space-time, cloud string.

1. Introduction

An interesting area of research is still the investigation of the early stages of the universe's formation and its origins. Phenomena that have not yet been seen or analyzed discover more about the universe's evolution. So, cosmologists have expressed a significant amount of interest in comprehending its future growth and understanding the universe's past, present, and future scenarios. But various perspectives are offered by various minds regarding the universe, so we have not yet reached a conclusion. Strong contended about its history and evolution confirmation. Our moral is therefore to conduct additional research responsibility to find the ultimate truth of cosmological phenomena as well as numerous other mysterious particles that have not yet been observed.

Alternative gravitational theories have generated a lot of interest in recent years. Because of the universe's late-time acceleration and the existence of dark matter and dark energy, modified theories of gravity have only recently been developed. Spatial homogeneous and anisotropic cosmological models play an important role in the description of the universe's massive scale behaviour, and such models have been widely studied by many authors in search of a relativistic picture of the early universe. The simplicity of the field equations and the relative ease with which Bianchi space times can be solved are useful in building models of spatially homogeneous and anisotropic cosmologies. Singh *et al.*[1] investigate the interaction of Bianchi type-I anisotropic cloud string cosmological model with electromagnetic field. Saha & Visinescu [2] studied Bianchi type-VI model with cosmic strings in presence of a magnetic field. Anisotropic

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Bianchi type-V I h perfect fluid cosmological models in a modified theory of gravity have been presented by Rao et al.[3] .

The most prominent gravitational modified theory is $f(T)$ gravity. Construction of viable modified teleparallel gravity models has developed as an alternative to general relativity, and in recent years, the cosmological applications of this theory have attracted considerable attention in the literature. Recently proposed modified gravity theories include *f(G), f(R,G)* and *f (R,T)*. By substituting the function of the Ricci scalar for R, Einstein-Hilbert action is modified. These theories are classified as *f(R)* theories of gravitation. Baojiu Li et al.[4] have explored Large-scale structure in $f(T)$ theory. Hatkar et al. [5] used the metric *f(R)* formalism to investigate the dark energy scenario. Katore et al. [6] explored FRW domain walls in the modified *f(G)* theory of gravitation. *f(T)* is another significant modified theory of gravitation, where T is a torsion scalar. In contrast to the curvature scalar R of standard general relativity, the torsion scalar T in the cosmological background does not include the time derivative of the Hubble parameter H.

In the framework of general relativity, tetrad components are derived using the Weitzenbock connection, whereas metric components are derived using the Levi-Civita connection. When compared to *f(R)* gravity, this feature provides a significant advantage in the reconstruction procedure. As a result, the teleparallel gravity scenario, which employs the transformation of the tetrad components to the metric components, replaces general relativity. To put it another way, in the teleparallel scenario, the curvature term R from general relativity is transformed into a torsion term *T*, and its modified form is transformed from T to *f(T)* by an arbitrary function in the associated action, known as the *f(T)*cosmology theory.

The strings are in fact hypothetical one-dimensional topological defects that occurred during phase transition from a temperature below a critical temperature in the early stages of the universe's evolution. The presence of strings in the early universe does not quarrel with the observations made presently. These strings interact with the gravitational field and also have stress energy. Even though strings are presently undetectable, their presence causes space-time anisotropy. The string can simultaneously describe the nature and basic structure of the early universe. Roy et al. [7] studied Bianchi type I cosmological models with perfect fluid and magnetic field. The Bianchi type-III charged fluid universe in the Brans-Dicke theory of gravitation was examined by Mete *et al*. [8]. Singh et al. [9] studied anisotropic cloud string cosmological model with Bianchi Type-I space-time in general relativity. Mete et al.[10] have discussed Bianchi type- VIo magnetized cosmological model in $f(R,T)$ theory of gravitation.

String cloud cosmologies for Bianchi type-III models with electromagnetic fields were examined by Tripathy et al. [11]. Bianchi type-I metric with massive string was presented by Pradhan *et al.*[12] in general relativity. The accelerating Bianchi type dark energy cosmological model with cosmic string in $f(T)$ gravity has been studied by Shekh et al. [13]. Dark energy, the accelerating universe, and exponential $f(T)$ gravity were all presented by Chirde et al.[14]. Houndjo *et al.*[15] studied cylindrical solutions in modified *f(T)* gravity. Chirde *et al.*[16] investigated anisotropic background for one-dimensional cosmic string in *f(T)* gravity. In this theory, gravitation is attributed to the torsion of a zero-curvature space-time, which acts as a force. Bianchi type-I spatially homogenous models, whose spatial sections are flat, are the most straightforward anisotropic models typically employed to describe the anisotropic effect. An advantage of adopting anisotropic models is their important role in the description of the early stage of universe. Bianchi type-I homogeneous and anisotropic space-time has been taken into consideration by several researchers. Katore *et al*.[17] studied plane symmetric cosmological models with perfect fluid and dark energy. Recently Nerkar et al. [18] studied dark energy Bianchi type-III cosmological models in $f(T)$ theory of gravity.

In this paper, we examine Bianchi type-VI$_0$ space-time with cloud sting within the framework of $f(T)$ gravity. We introduce preliminary definitions of $f(T)$ gravity in Section 2. In Section 3, we explore the field equations together with solutions and some physical and kinematic parameters. The conclusions are given in Section 4.

2. Preliminary definitions and equation of motion of $f(T)$ gravity

In this section, we provide a concise explanation of $f(T)$ gravity and a thorough derivation of its field equations. Let us define the Greek and Latin notations of the Latin subscript as those connected to the space-time coordinates and the tetrad field, respectively. We can define the line element for a general space-time metric as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu , \tag{1}$$

where $g_{\mu\nu}$ are the components of the metric which is symmetric and possesses ten degrees of freedom. The theory can be expressed either in space-time or in tangent space, allowing us to rewrite the line element that can be transformed into the tetrad described by Minkowski (which represents the dynamic fields of the theory) as follows.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ij}\theta^i\theta^j \tag{2}$$

$$dx^\mu = e_i^\mu \theta^i , dx^\nu = e_i^\nu \theta^j , \tag{3}$$

where η_{ij} is a metric on Minkowski space-time and $\eta_{ij} = diag[1,-1,-1,-1]$ and $e_i^\mu e_\nu^i = \delta_\nu^\mu$ or $e_i^\mu e_\mu^j = \delta_i^j$. The Square root of metric determinant is given by $\sqrt{-g} = \det[e_\mu^i] = e$ and the matrix e_μ^α are called tetrads and represent the dynamic fields of the theory. The Weitzenbocks connection components for a manifold where the Riemann tensor part without the torsion terms is null (contribution of the Levi-Civita connection) and only the non-zero torsion terms exist, are defined as follows

$$\Gamma_{\mu\nu}^\alpha = e_i^\alpha \partial_\nu e_\mu^i = -e_\mu^i \partial_\nu e_i^\alpha \tag{4}$$

which has a zero curvature but nonzero torsion. The main geometrical objects of the space-time are constructed from this connection. Through the connection, the components of the tensor torsion are defined by the anti-symmetric part of this connection as

$$T_{\mu\nu}^\alpha = \Gamma_{\mu\nu}^\alpha - \Gamma_{\nu\mu}^\alpha = e_i^\alpha (\partial_\mu e_\nu^i - \partial_\nu e_\mu^i) \tag{5}$$

We also define the components of the so-called con-torsion tensor as

$$K_{\alpha}^{\mu\nu} = (-\tfrac{1}{2})(T_{\alpha}^{\mu\nu} - T_{\alpha}^{\nu\mu} - T_{\alpha}^{\mu\nu}) \tag{6}$$

In order to make more clear the definition of the scalar equivalent to the curvature scalar of RG, we first define a new tensor $S_{\alpha}^{\mu\nu}$, constructed from the components of the torsion and contorsion tensors as

$$S_{\alpha}^{\mu\nu} = \tfrac{1}{2}(K_{\alpha}^{\mu\nu} - \delta_{\alpha}^{\mu} T_{\beta}^{\beta\nu} - \delta_{\varepsilon}^{\nu} T_{\beta}^{\beta\mu}) \tag{7}$$

Now, we are able to construct a contraction which is equivalent to the scalar curvature in GR. We define then the torsion scalar as

$$T = T_{\mu\nu}^{\alpha} S_{\alpha}^{\mu\nu} \tag{8}$$

Now, we define the action by generalizing the TG i.e. $f(T)$ theory as

$$S = \int [T + f(T) + L_{Matter}] e d^4 x \tag{9}$$

Here, $f(T)$denotes an algebraic function of the torsion scalar T. Making the functional variation of the action (9) with respect to the tetrads, we get the following equations of motion

$$S_{\mu}^{\nu\rho} \partial_{\rho} T f_{TT} + [e^{-1} e_{\mu}^{i} \partial_{\rho} (e e_{i}^{\alpha} S_{\mu}^{\nu\rho}) + T_{\lambda\mu}^{\alpha} S_{\alpha}^{\nu\lambda}] f_T + \tfrac{1}{4} \delta_{\mu}^{\nu} f = 4\pi T_{\mu}^{\nu} \tag{10}$$

The field equation (10) is written in terms of the tetrad and partial derivatives and appears very different from Einstein's equation, where, T_{μ}^{ν} is the energy momentum tensor, $f_T = {df(T)}/{dT}$, $f_{TT} = {d^2 f(T)}/{d\,T^2}$ and by setting $f(T) = a_0 = \text{constant}$, the equations of motion (10) are the same as that of the teleparallel gravity with a cosmological constant, and this is dynamically equivalent to the GR. These equations clearly depend on the choice made for the set of tetrads.

3. Field equations for Bianchi Type – VI$_0$ Model

We consider homogeneous anisotropic Bianchi type-VI$_0$ line element, given by

$$ds^2 = dt^2 - A^2(t) dx^2 - B^2(t) e^{2x} dy^2 - C^2(t) e^{-2x} dz^2 , \tag{11}$$

where, the scale factors A,B and C are functions of cosmic time t only.
Let us choose the following set of diagonal tetrads related to the metric (11)

$$[e_{\mu}^{\nu}] = diag[1, A, B, C] \tag{12}$$

The determinant of the matrix (11) is

$$e = ABC \tag{13}$$

The components of the tensor torsion (5) for the tetrads (11) are given by

$$T_{01}^{1} = \frac{\dot{A}}{A},\ T_{02}^{2} = \frac{\dot{B}}{B},\ T_{03}^{3} = \frac{\dot{C}}{C},\ T_{12}^{2} = -1,\ T_{13}^{3} = 1 \tag{14}$$

The components of the tensor $S_{\alpha}^{\mu\nu}$, in (7), are given by

$$S_{2}^{21} = \frac{1}{2},\ S_{1}^{10} = \frac{1}{2}\left(\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\right),\ S_{2}^{20} = \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{C}}{C}\right),\ S_{3}^{31} = -\frac{1}{2},\ S_{3}^{30} = \frac{1}{2}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{15}$$

The corresponding torsion scalar (8) is given by

$$T=-2\left(2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}-1\right) \tag{16}$$

Here we take a more general cloud sting tensor in the following form

$$T_{ij}=\rho u_{\mu}u^{\upsilon}-\lambda x_{\mu}x^{\upsilon} \tag{17}$$

$$T_1^1=\lambda, T_2^2=0, T_3^3=0, T_0^0=\rho, T_\mu^\nu=0 \;\; for\; \nu\neq\mu\;,$$

where u^ν is the four-velocity vector while ρ and λ are the energy density and tension density of the cloud sting respectively.

Now, the field equations for cloud string and Bianchi type-VI_0 space-time, in the framework of $f(T)$ gravity, are obtained as

$$f+4f_T\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{C}}{C}\frac{\dot{B}}{B}\right)=16\pi T_0^0 \tag{18}$$

$$f+2f_T\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+2\frac{\dot{C}}{C}\frac{\dot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\ddot{B}}{B}-2\right)+2\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\dot{T}f_{TT}=16\pi T_1^1 \tag{19}$$

$$f+2f_T\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B}+2\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{C}}{C}\frac{\dot{B}}{B}+\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\dot{T}f_{TT}=16\pi T_2^2 \tag{20}$$

$$f+2f_T\left(2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{C}}{C}\frac{\dot{B}}{B}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{T}f_{TT}=16\pi T_3^3 \tag{21}$$

$$\frac{1}{2}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)f_T=0\quad , \tag{22}$$

where, the dot (.) denotes the derivative with respect to time *t*.
From eq. (22), we get $B=kC$ without loss of generality we take *k*=1 so that we have

$$B=C \tag{23}$$

Using (23), field equations (18) to (21) can reduce to

$$f+4f_T\left(2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}\right)=16\pi(\rho) \tag{24}$$

$$f+4f_T\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{\ddot{B}}{B}-1\right)+4\left(\frac{\dot{B}}{B}\right)\dot{T}f_{TT}=16\pi(\lambda) \tag{25}$$

$$f+2f_T\left(3\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}\right)+2\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\dot{T}f_{TT}=0\,. \tag{26}$$

Here we have three differential equations with five unknowns namelyA, B, ρ, λ, f. The solution of these equations is discussed in next section. In the following, we define some important physical quantities of the space time

We assume the analytic relation between the metric coefficients as

$$A = B^n \tag{27}$$

We define some kinematical space-time quantities. Average scale factor (a) and volume (V) respectively as

$$a = \sqrt[3]{AB^2}\ ,\quad V = a^3 \tag{28}$$

The generalized mean Hubble parameter (H), which describes the volumetric expansion rate of the universe, is

$$H = \tfrac{1}{3}(H_1 + H_2 + H_3)\ , \tag{29}$$

where, H_1 , H_2 , H_3 are the directional Hubble parameters.

Using Eqns. (28) and (29), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a}\ . \tag{30}$$

We discuss the mean anisotropy parameter (A_m) of the form to analyze whether the model approaches isotropy or not.

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H} - 1\right)^2 \tag{31}$$

The expansion scalar (θ) and the shear scalar (σ^2) are defined as

$$\theta = u^{\mu}_{;\mu} = 3H \tag{32}$$

$$\sigma^2 = \frac{3}{2}A_m H^2 \tag{33}$$

Physical and kinematical parameters

Model I:

We consider the value of the average scale factor corresponding to the model of the universe as

$$a = \sinh mt \tag{34}$$

The value of the deceleration parameter is given for the mean scale factor in Eqn. (34) as

$$q = -1 + \frac{1}{\cosh^2 mt} \tag{35}$$

It is important to keep in mind that the average scale factor *a(t)*, must be known in order to calculate the value of metric functions. The deceleration parameter and the Hubble parameter are essential for building cosmological models since they describe how the cosmos behaves. Recent findings indicate that the cosmos was decelerating in the past and is accelerating right now. As a result, it is typically assumed that the deceleration parameter's value will take both a constant and a time-dependent form. Many authors have suggested different time-dependent deceleration parameter forms and have found various average scale factor forms for the model.

The associated metric coefficients A and B for this model become

$$A = (\sinh mt)^{\frac{3n}{n+2}} \tag{36}$$

$$B = (\sinh mt)^{\frac{3}{n+2}} \tag{37}$$

Using Eqns. (36) and (37), we get

$$ds^2 = dt^2 - (\sinh mt)^{\frac{3n}{n+2}} dx^2 - (\sinh mt)^{\frac{3}{n+2}} (dy^2 + dz^2) \tag{38}$$

The Torsion scalar (T) becomes

$$T = 2 - \frac{18(2n+1)m^2 \cosh^2 mt}{(n+2)^2} \tag{39}$$

The spatial volume (V)becomes

$$V = (\sinh mt)^3 \tag{40}$$

The mean Hubble parameter (H) and the expansion scalar (θ) turn out to be

$$H = m \coth mt \tag{41}$$

$$\theta = 3m \coth mt \tag{42}$$

The average scale factor and spatial volume disappear with time $t \to 0$. As time $t \to 0$, the model begins to expand with a zero volume. When time increases expansion scalar decreases and as time $t \to 0$, the mean Hubble parameter is initially large and decreases with increasing time t. The expansion scalar θ decreases as time $t \to \infty$ shown in Fig.1. Which indicates that universe is expanding with increase with time t.

The mean anisotropy parameter (A_m) and shear scalar (σ^2) are given by

$$A_m = \frac{2(n-1)^2}{(n+2)^2} \tag{43}$$

$$\sigma^2 = \frac{3(n-1)^2 m^2 \coth^2 mt}{(n+2)^2} \tag{44}$$

It has been found that the spatial volume vanishes at starting time $t = 0$, expands with time, and becomes infinitely massive at $t = \infty$ .In comparison to the shear scalar, which is time dependent and decreases with time as the universe expands, the mean anisotropy parameter is independent of time t and remains constant throughout the universe's evolution from early to infinite expansion. This indicates how the universe is expanding with the flow of time while slowing its rate of growth to a constant value, showing how the universe began to expand at an infinite rate.

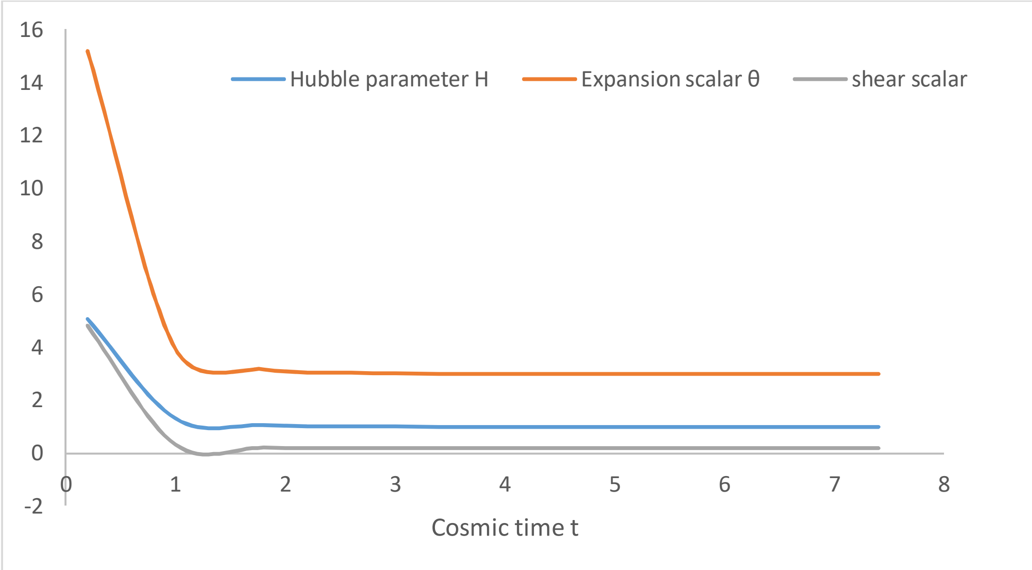

Fig.1. Graphical representation of Hubble's parameter, expansion scalar and shear scalar versus cosmic time t.

The general exact solution for a viable $f(T)$model with a quadratic correction term, $f(T)=\eta(-T)^{\alpha}$ will be derived in this section. Where η is constant. In particular, considering the basic and usual ansatz $f(T)=\eta(-T)^{\alpha}$ is a good approximation in all realistic cases. Chirde and Hatkar [19] adopted $f(T)$ for the investigation of Bianchi type I cosmological model with perfect fluid and string in *f (T)* theory of gravitation.

$$f(T)=\eta(-T)^{\alpha}$$

$$f_T=\eta\alpha(-T)^{\alpha-1}$$

$$f_{TT}=\eta\alpha(\alpha-1)(-T)^{\alpha-2}$$

The value of energy density and tension density of cloud sting become,

$$\rho=\frac{\eta}{16\pi}\left\{[-(2-k_1\coth^2 mt)]^{\alpha}-k_2[-(2-k_1\coth^2 mt)]^{\alpha-1}\coth^2 mt\right\} \tag{45}$$

$$\lambda=\frac{\eta}{16\pi}[-(2-k_1\coth^2 mt)]^{\alpha}\left\{\begin{aligned}&1-4\alpha[-(2-k_1\coth^2 mt)]^{-1}(-k_2\frac{1}{\sinh^2 mt}+3k_2\coth^2 mt-1)\\&+k_4[-(2-k_1\coth^2 mt)]^{-2}\frac{\coth^2 mt}{\sinh^2 mt}\end{aligned}\right\} \tag{46}$$

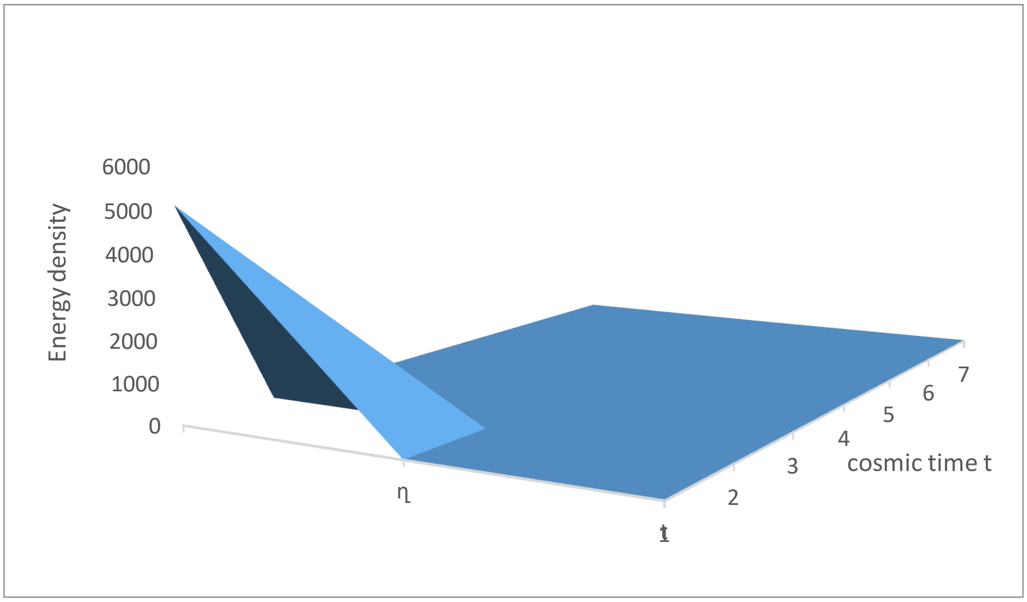

Fig.2. Energy density versus cosmic time

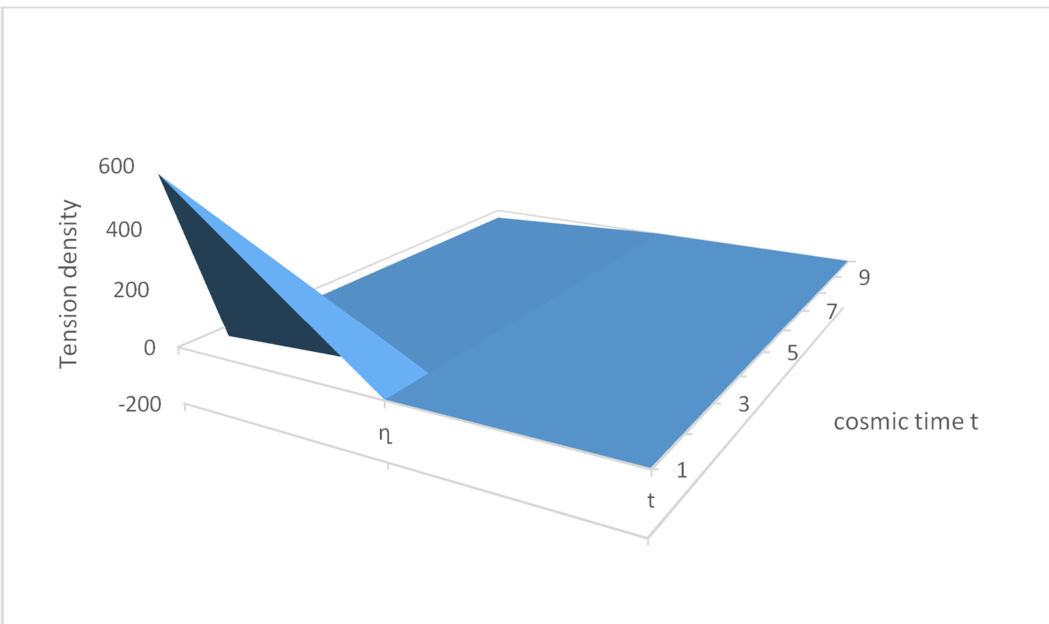

Fig.3. Tension density versus cosmic time

where $k_1 = \dfrac{18m^2(2n+1)}{(n+2)^2}$, $k_2 = 72\alpha m^2(n+2)$, $k_3 = \dfrac{3m^2}{(n+2)}$

$$k_4 = \frac{432\alpha(\alpha-1)m^4(2n+1)}{(n+2)^3}$$

The energy density ρ is always positive and decreases as cosmic time t increases. The universe has an infinitely large energy density at the beginning, but it decreases with expansion and becomes null at very large expansion, shown in fig.2. It is clear that tension density λ initially is very high and gradually it decreases with increase of time, shown in fig.3.

Model II:

We consider the value of the average scale factor corresponding to the model of the Universe as

$$a = t^b \tag{47}$$

The value of the deceleration parameter is given for the mean scale factor in Eqn. (47) as

$$q = -1 + \frac{1}{b}. \tag{48}$$

When b>1, we have q< -1 the universe is accelerating for b> 1. The sign of deceleration parameter indicates whether the universe is accelerating or decelerating. The negative sign of q indicates accelerating universe whereas positive sign of q indicates decelerating universe. Here q negative, i.e. the universe is accelerating. Katore & Baxi [20] studied Stability of Kaluza–Klein holographic dark energy cosmological models in *f (R)* theory of gravitation.
The associated metric coefficients A and B for this model become

$$A = t^{\frac{3nb}{n+2}} \tag{49}$$

$$B = t^{\frac{3b}{n+2}} \tag{50}$$

Using Eqns. (49)and (50), we get,

$$ds^2 = dt^2 - t^{\frac{3nb}{n+2}} dx^2 - t^{\frac{3b}{n+2}} (e^{2x} dy^2 + e^{-2x} dz^2) \tag{51}$$

The Torsion scalar (T) becomes

$$T = 2 - \frac{18(2n+1)b^2}{(n+2)^2 t^2} \tag{52}$$

The spatial volume (V)becomes

$$V = a^3 = t^{3b} \tag{53}$$

The mean Hubble parameter (H) and the expansion scalar (θ) turn out to be

$$H = \frac{b}{t} \tag{54}$$

$$\theta = 3H = \frac{3b}{t} \tag{55}$$

The spatial volume disappears with time $t \to 0$, as seen. At time $t \to 0$, the model begins to expand with a zero volume.The mean Hubble parameter is initially large and zero at time $t \to \infty$.The expansion scalar $\theta \to 0$ as time $t \to \infty$ as shown in fig.4.Which indicates that universe is expanding with increase with time.

The mean anisotropy parameter (A_m) and shear scalar $(\sigma^2)$are given by

$$A_m = \frac{2(n-1)^2}{(n+2)^2} \tag{56}$$

$$\sigma^2 = 3\frac{(n-1)^2 b^2}{(n+2)^2 t^2} \tag{57}$$

It has been found that the spatial volume vanishes at starting time $t = 0$, expands with time, and becomes infinitely massive at $t = \infty$. In comparison to the shear scalar, which is time dependent and decreases with time as the universe expands, the mean anisotropy parameter is independent of time t and remains constant throughout the universe's evolution from early to infinite expansion. This indicates how the universe is expanding with the flow of time while slowing its rate of growth to a constant value, showing how the universe began to expand at an infinite rate.

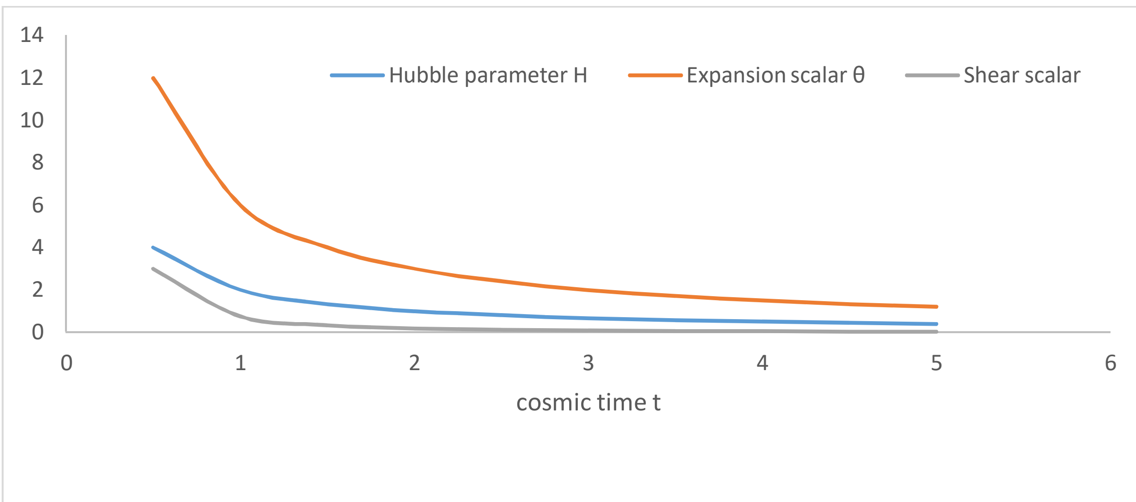

Fig.4. Graphical representation of Hubble's parameter, expansion scalar and shear scalar versus cosmic time *t*.

The value of energy density and tension density become,

$$\rho = \frac{\eta}{16\pi}[-(2 - k_1 \frac{1}{t^2})]^\alpha \left[1 - \frac{2\alpha k_1}{2t^2 - k_1}\right] \tag{58}$$

$$\lambda = \frac{\eta}{16\pi}[-(2-k_1\frac{1}{t^2})]^\alpha \left[1 - \frac{4\alpha(k_2 - t^2)}{(2t^2 - k_1)} + \frac{k_3}{(2t^2 - k_1)^2}\right] \quad , \tag{59}$$

where $k_1 = \dfrac{18b^2(2n+1)}{(n+2)^2}$, $k_2 = \dfrac{-3b - 6b + 18b^2 + 9nb^2}{(n+2)^2}$,

$$k_3 = \frac{432\alpha(\alpha-1)b^3(2n+1)}{(n+2)^3}$$

The behavior of the energy density ρ for an appropriate choice of constants in fig. 5. The energy density is decreasing function of time t. It is clear that sting tension density λ initially is very high and gradually it decreases with increase of time shown in fig.6.

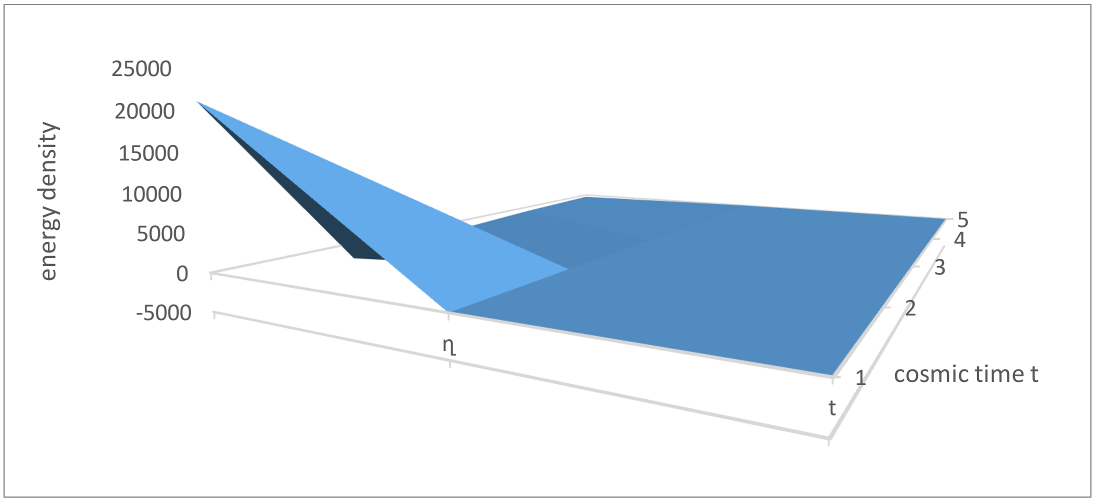

Fig.5. Energy density versus cosmic time

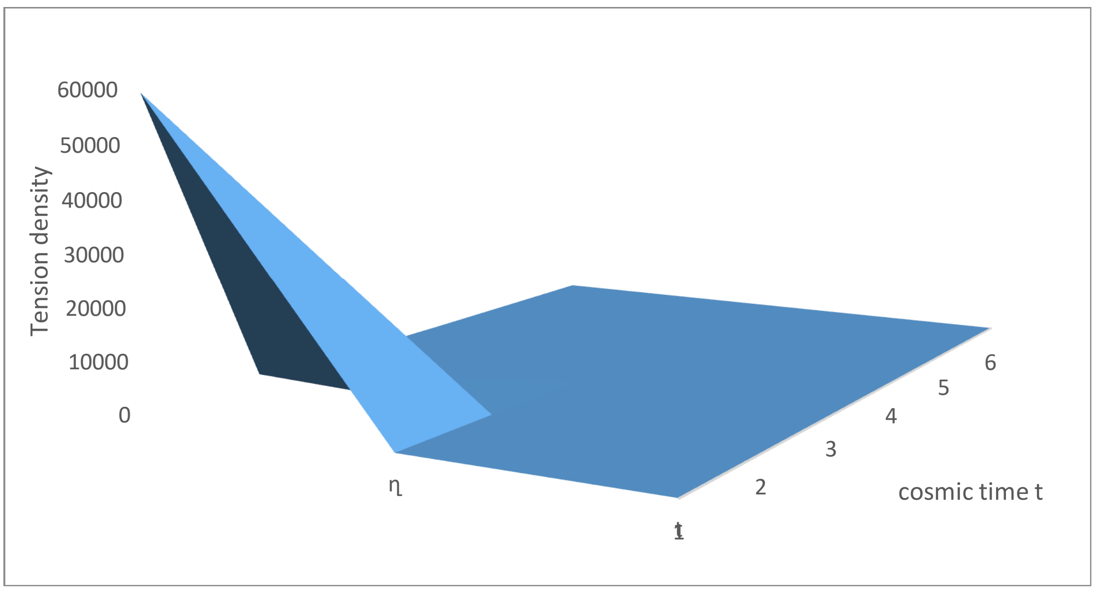

Fig.6. Tension density versus cosmic time

4. Conclusions

In this paper, we have presented the solutions of Bianchi type-VI_0 cosmological model in presence of cosmic string in $f(T)$ theory of gravity. For this purpose we use scalar factors $a = t^b$,$a = sinhmt$ and functional form of $f(T)$ such as $f(T) = \eta(-T)^\alpha$. We have evaluated some physical parameters for this solution such as H,θ,A_m, σ^2. These physical parameters, from both the models, we find that energy density is very large initially and at latter time it vanishes. In both models I & II, the string tension density shows same behavior, initially, it is very high and gradually it decreases with increase of time.

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