

Exploration

One-Note-Samba Approach to Cosmology

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Abstract

Inspired by One Note Samba, a standard jazz repertoire, we present an outline of Bose-Einstein Condensate Cosmology. Although this approach seems awkward and a bit off the wall at first glance, it is not impossible to connect altogether BEC, Scalar Field Cosmology and Feshbach Resonance with Ermakov-Pinney equation. We also briefly discuss possible link with our previous paper which describes Newtonian Universe with Vortex in terms of Ermakov equation.

Keywords: Cosmology, Bose-Einstein condensate, scalar field, Feshbach resonance, Ermakov-Pinney equation, Newtonian universe, vortex.

Introduction

From time to time, it is often found useful to come up with a new approach in cosmology studies, in order to seek a new insight from where we can develop and take further step.

For instance, it is known that flat spacetime cosmology can explain many cosmology phenomena (see for instance Narlikar & Arp [30]). But then, if we know that there is quite high likelihood that our Universe can be modeled as flat spacetime, then what else to be done?

In this occasion, allow us to put forth an argument that our Universe has remarkable similarity with a macroscale Bose-Einstein condensate, especially on the grounds: a. the CMBR temperature is found to be as low as 2.73^0 Kelvin, therefore it may indicate a low temperature physics model of Universe (see G. Volovik [31]), and b. recent discovery of “black hole” seems remarkably similar to a vortex ring of BEC experiment (it shows a dark spot circled with a white ring).

All in all, although we admit that this approach seems awkward and a bit off the wall at first glance, it is not impossible to connect altogether BEC, Scalar Field Cosmology and Feshbach Resonance with Ermakov-Pinney equation. We also discuss shortly possible link with our previous paper, where we describe Newtonian Universe with Vortex in terms of Ermakov equation [5].

We submit to call this approach: “*one note Samba*,” i.e. starting with a very simple premise (BEC) you arrive at a model of the entire Universe.

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Lidsey's BEC-Cosmodynamics Correspondence

According to Hawkins & Lidsey [2], the dynamics of cosmologies sourced by a mixture of perfect fluids and self-interacting scalar fields are described by the non-linear, Ermakov-Pinney equation. The general solution of this equation can be expressed in terms of particular solutions to a related, linear differential equation. In general, an Ermakov system is a pair of coupled, second-order, non-linear ordinary differential equations (ODEs) and such systems often arise in studies of nonlinear optics [12], nonlinear elasticity, molecular structures.

They developed an analytical approach to models of this type by expressing the cosmological field equations in terms of an Ermakov system. In the one-dimensional case, the two equations decouple and the system reduces to a single equation known as the Ermakov-Pinney equation [2]:

$$\frac{d^2b}{d\tau^2} + Q(\tau)b = \frac{\lambda}{b^3} \quad (1)$$

where Q is an arbitrary function of τ and λ is a constant. Equation (1) is sometimes referred to as the Milne-Pinney equation. They also showed that the field equations for a spatially flat, Friedmann-Robertson-Walker (FRW) universe with a scalar field and perfect fluid matter source reduce to Ermakov-Pinney equation. To summarize, the dynamics of a pure scalar field cosmology is determined by a one-dimensional oscillator equation with a time-dependent frequency. [2]

In his subsequent paper, Lidsey managed to show that there is dynamical correspondence between positively curved, isotropic, perfect fluid cosmologies and quasi-two-dimensional, harmonically trapped Bose-Einstein condensates by mapping the equations of motion for both systems onto the one-dimensional Ermakov system.[1]

He developed that connection based on *analogies between various condensed matter systems and different branches of gravitational physics which have been developed in recent years*. For example, the propagation of acoustic waves in an irrotational, inviscid, barotropic fluid is formally equivalent to that of a massless scalar field on a curved, Lorentzian spacetime. Furthermore, it is possible to model a black hole acoustically in terms of supersonic fluid flow and, in principle, quantum effects associated with black hole event horizons may then be studied within the context of condensed matter configurations.[1]

Identifying such a link is quite significant because if it can be shown that there exists dynamical correspondence between isotropic, four-dimensional cosmological models and harmonically

trapped, quasi-two-dimensional Bose-Einstein condensates, then there is a big hope to do simulation or cosmology experiments in lab. The correspondence arises because the equations of motion for both systems can be mapped onto the one-dimensional Ermakov system.

In that paper, Lidsey showed that the dynamics of a positively curved ($k > 0$) FRW cosmology can be modeled in terms of a harmonically trapped Bose-Einstein condensate when cosmic time, τ is related to ‘laboratory’ time, t . [1]

To summarize our discussion thus far, it can be shown that positively curved, perfect fluid FRW cosmologies can be modeled dynamically in terms of quasi-two-dimensional Bose-Einstein condensates, where there exists a one-to-one correspondence between the type of matter in the universe and the functional form of the time-dependent trapping potential of the condensate. The physical properties of the wavefunction can be identified with the fundamental cosmological parameters. [1]

The Pinney equation corresponding to FRW cosmology can be written as follows [1]:

$$\frac{d^2 a}{dt^2} + \left(\frac{d\phi}{dt}\right)^2 a = \frac{k}{a^3} \quad (2)$$

A key assumption that was made in establishing the correspondence between the condensate and cosmological systems was that the dynamics of the condensate wavefunction can be described in terms of the Gross-Pitaevskii equation at each moment of time, i.e., that the configuration reacts instantaneously to changes in the trapping potential and scattering length of the atomic interactions. If this assumption is to remain valid, the majority of the atoms must remain in the condensate state (mean-field approximation) and the particle density and scattering length must be sufficiently small (dilute gas approximation) [1].

Nevertheless, one advantage of establishing correspondences between cosmology and condensed matter physics through Ermakov systems is that insight into the hidden symmetries of the two systems may be uncovered.

Subsequent work by Herring et al. revisit the topic of two-dimensional Bose-Einstein condensates under the influence of time-dependent magnetic confinement and time-dependent scattering length. A moment approach reduces the examination of moments of the wavefunction (in particular, of its width) to an Ermakov-Pinney (EP) ordinary differential equation (ODE). They discussed Feshbach resonance managed BEC and how EP equation connects with the case of anisotropic scalar field cosmologies [3].

There is also a more recent report by D'Ambroise and Williams showing that there is also dynamic correspondence not only to FLRW but also Bianchi I cosmologies and BEC system in arbitrary dimension, especially when a cosmological constant is present [4].

Comparison with Newtonian Dynamics Model

In a previous paper [5], we presented a numerical solution of Newtonian Universe with vortex; see also [6][7]. Now we will present a more detailed account of our model.

A physical model of turbulence-generated sound for early Universe

Our discussion starts from the fundamental question: how can we include the rotation in early Universe model? After answering that question, we will discuss how “turbulence-generated sound” can be put into a mathematical model for the early Universe. We are aware that the notion of turbulence-generated sound is not new term at all especially in aerodynamics, but the term is rarely used in cosmology until now. We shall show that 3D Navier-Stokes will lead to non-linear acoustics models, which means that a turbulence/storm can generate sound wave.

a. How can we include rotation in early Universe model?

It has been known for long time that most of the existing cosmology models have singularity problem. Cosmological singularity has been a consequence of excessive symmetry of flow, such as “Hubble’s law”. More realistic one is suggested, based on Newtonian cosmology model but here we include the vortical-rotational effect of the whole Universe.

As shown in previous paper, we derived an Ermakov-type equation following Nurgaliev [26][27].

After he proceeds with some initial assumptions, Nurgaliev obtained a new simple local cosmological equation:[8][9]

$$\dot{H} + H^2 = \omega^2 + \frac{4\pi G}{3} \rho, \quad (3)$$

where $\dot{H} = dH / dt$.

The angular momentum conservation law $\omega R^2 = \text{const} = K$ and the mass conservation law $(4\pi/3)\rho R^3 = \text{const} = M$ makes equation (3) solvable:[26]

$$\dot{H} + H^2 = \frac{K^2}{R^4} - \frac{GM}{R^3}, \quad (4)$$

or

$$\ddot{R} = \frac{K^2}{R^3} - \frac{GM}{R^2}.$$

(5)

Equation (5) may be written as Ermakov-type nonlinear equation as follows;

$$\ddot{R} + \frac{GM}{R^2} = \frac{K^2}{R^3}. \tag{6}$$

Nurgaliev tried to integrate equation (5), but we solved the above equation numerically. The results are as follows: First, we rewrite this equation by replacing $GM=A$, $K^2=B$, so we get an expression of Ermakov equation:[26]

$$\ddot{R} + \frac{A}{R^2} = \frac{B}{R^3}. \tag{7}$$

As with what Nurgaliev did, we also tried different sets of A and B values, as follows:

a. A and $B < 0$

A

$B=-10$;

$ODE=x''[t]+A/x[t]^2-B/x[t]^3==0$;

$sol=NDSolve[\{ODE,x[0]==1,x'[0]==1\},x[t],\{t,-10,10\}]$

$Plot[x[t]/.sol,\{t,-10,10\}]$

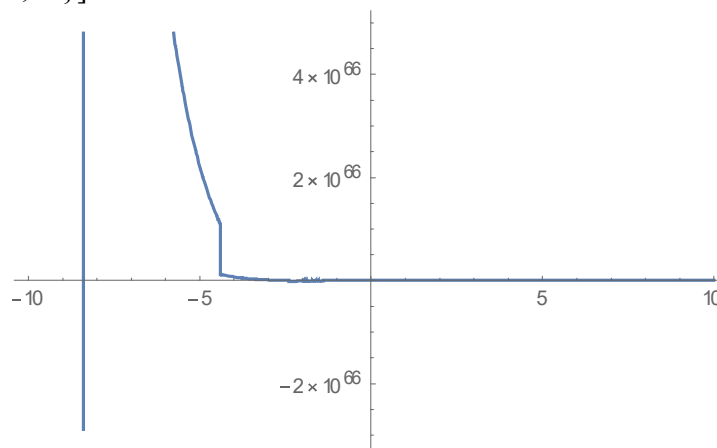


Figure 1. Plot of Ermakov-type solution for $A=-10$, $B=-10$ [5]

b. $A > 0$, $B < 0$

$A=1$;

$B=-10$;

$ODE=x''[t]+A/x[t]^2-B/x[t]^3==0$;

$sol=NDSolve[\{ODE,x[0]==1,x'[0]==1\},x[t],\{t,-10,10\}]$

$Plot[x[t]/.sol,\{t,-10,10\}]$

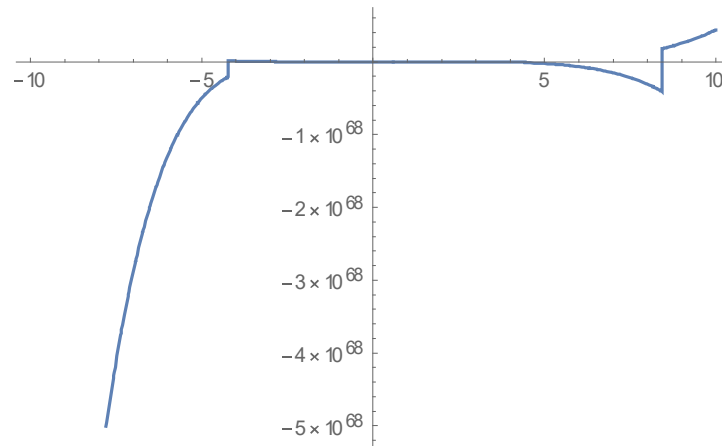


Figure 2. Plot of Ermakov-type solution for $A=1$, $B=-10$ [5]

From the above numerical experiments, we conclude that the evolution of the Universe depends on the constants involved, especially on the rotational-vortex structure of the Universe. This needs to be investigated in more detailed for sure.

One conclusion that we may derive especially from Figure 2, is that our computational simulation suggests that it is possible to consider that the Universe has existed for long time in prolonged stagnation period, then suddenly it burst out from *empty and formless* (Gen. 1:2), to take its current shape with accelerated expansion.

As an implication, we may arrive at a precise model of flattening velocity of galaxies without having to invoke *ad-hoc* assumptions such as dark matter.

Therefore, it is perhaps noteworthy to discuss briefly a simple model of galaxies based on a postulate of turbulence vortices which govern the galaxy dynamics. The result of Vastistas' model equation can yield prediction which is close to observation, as shown in the following diagram:[10]

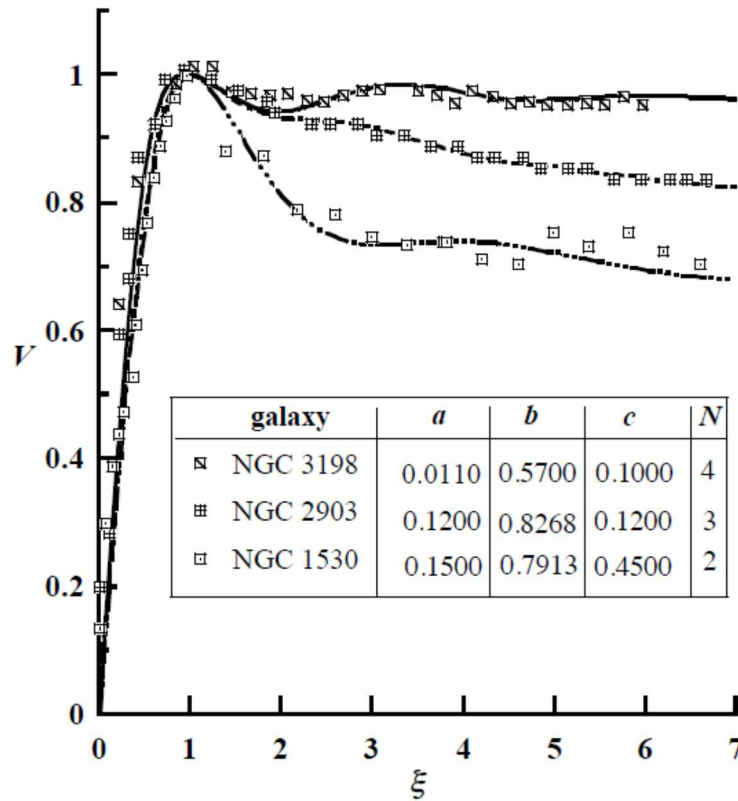


Figure 3. From Vatisas [10]

Therefore it appears possible to model galaxies without invoking numerous *ad hoc* assumptions such as *dark matter*, once we accept the existence of turbulent interstellar medium. The Vatisas model is also governed by Navier-Stokes equations, see for instance [10].

b. *How “turbulence-generated sound” can be put into a mathematical model for the early Universe*

We are aware that the notion of turbulence-generated sound is not new term at all especially in aerodynamics, but the term is rarely used in cosmology until now. We will consider some papers where it can be shown that 3D Navier-Stokes will lead to *non-linear acoustics* models, which means that a turbulence/storm can generate sound wave.

In this section we consider only two approaches:

- Shugaev-Cherkasov-Solenaya’s model: They investigate acoustic radiation emitted by three-dimensional (3D) vortex rings in air on the basis of the unsteady Navier–Stokes equations. Power series expansions of the unknown functions with respect to the initial vorticity which is supposed to be small are used. In such a manner the system of the

Navier–Stokes equations is reduced to a parabolic system with constant coefficients at high derivatives. [11]

- Rozanova-Pierrat’s Kuznetsov equation: she analysed the existing derivation of the models of non-linear acoustics such as the Kuznetsov equation, the NPE equation and the KZK equation. The technique of introducing a corrector in the derivation ansatz allows to consider the solutions of these equations as approximations of the solution of the initial system (a compressible Navier-Stokes/Euler system). The direct derivation shows that the Kuznetsov equation is the first order approximation of the Navier-Stokes system, the KZK and NPE equations are the first order approximations of the Kuznetsov equation and the second order approximations of the Navier-Stokes system. [12]

Remark on Neutrosophic Logic perspective and implications

It seems obvious, how this new scenario is quite in agreement with Kant’s idea that it is possible that the Universe has both finite history in the past and also eternal background (our new term: “*time before time*”); see [6]. We also discussed how such a mixed view can be modelled by introducing rotation in the early universe; see in particular Fig. 2.

Now there is an immediate question: Is this new look at the origin of Universe justifiable logically, or is it merely a compromised solution?

So, in this chapter we will review *Neutrosophic Logic*, a new theory developed in recent decades by one of these authors (FS). In this context, allow us to argue in favor of Neutrosophic logic as one basic postulate, in lieu of the Aristotle logic which creates many problems in real world.

In Neutrosophy, we can connect an idea with its opposite idea and with its neutral idea and get common parts, i.e. $\langle A \rangle \wedge \langle \text{non}A \rangle = \text{nonempty set}$. The common part of the uncommon things! It is true/real... paradox. From neutrosophy, all started: neutrosophic logic, neutrosophic set, neutrosophic probability, neutrosophic statistics, neutrosophic measure, neutrosophic physics, neutrosophic algebraic structures etc.

It is true in restricted case, i.e. the Hegelian dialectics considers only the dynamics of opposites ($\langle A \rangle$ and $\langle \text{anti}A \rangle$), but in our everyday life, not only the opposites interact, but the neutrals $\langle \text{neut}A \rangle$ between them too. For example: you fight with a man (so you both are the opposites). But neutral people around both of you (especially the police) interfere to reconcile both of you. Neutrosophy considers the dynamics of opposites and their neutrals.

So, neutrosophy means that: $\langle A \rangle$, $\langle \text{anti}A \rangle$ (the opposite of $\langle A \rangle$), and $\langle \text{neut}A \rangle$ (the neutrals between $\langle A \rangle$ and $\langle \text{anti}A \rangle$) interact among themselves. A neutrosophic set is characterized by a truth-membership function (T), an indeterminacy-membership function (I), and a falsity-membership function (F), where T, I, F are subsets of the unit interval [0, 1].

As particular cases we have: single-valued neutrosophic set {when T, I, F are crisp numbers in $[0, 1]$ }, and interval-valued neutrosophic set {when T, I, F are intervals included in $[0, 1]$ }. Neutrosophic Set is a powerful structure in expressing indeterminate, vague, incomplete *and* inconsistent information. See [16]-[18].

To summarize, Neutrosophic Logic studies the dynamics of opposites and neutralities. And from this viewpoint, we can understand that it is indeed a real possibility that the Universe has both initial start (creation) but with eternal background. This is exactly the picture we got after our closer look at Gen. 1:1-2 as discussed in the above section.

In other words, our proposed term of Kantian “*time before time*” has sufficient logical background, especially in turbulence Universe model. This new interpretation of cosmic dynamics can be considered as Neutrosophic Logic application in cosmology studies, see also our previous article [32].

In the next section, we will consider some advantages of this new model of Universe.

Advantages of our Turbulence Universe model

Now, allow us to discuss some advantages of the proposed turbulence cosmology view over the Lemaitre’s primeval atom (which is the basis of Standard Model Cosmology); see [13]-[15].

a. Avoid inflationary scheme.

It is known that inflationary models were proposed by Alan Guth et al. (see [19][20]), in order to explain certain difficulties in the Big Bang scenario. But some cosmology experts such as Hollands & Wald has raised some difficulties with inflationary model, as follows:

“We argue that the explanations provided by inflation for the homogeneity, isotropy, and flatness of our universe are not satisfactory, and that a proper explanation of these features will require a much deeper understanding of the initial state of our universe.”[21]

In our diagram plot above, it is clear that an early rotation model can explain why the Universe can burst out into creation in a very short period, without invoking ad hoc postulate such as inflation model.

b. Explain the observed late accelerated expansion.

As far as we know, one of the earliest models which gave prediction of accelerated expanding Universe is Carmeli’s Cosmological General Relativity.[23]

But it has been shown by Green & Wald that for the large scale structures of the Universe, Newtonian model can give similar results compared to general relativity picture.[22]

Furthermore, it seems that there is no quite clear arguments why we should accept Carmeli use of 5D metric model (*space-time-velocity metric*). In the meantime, in our rotating Universe model, we do not invoke *ad hoc* dimension into the metric.

- c. Explain inhomogeneity, breeding galaxies etc.

Astronomers have known for long time, that the Universe is not homogeneous and isotropic as in the usual model. It contains of inhomogeneity, irregularities, clumpiness, voids, filaments etc., which indicate complex structures. Such inhomogeneous structures may be better modelled in terms of turbulence model such as Navier-Stokes equations, see also our early papers [7][8], also [10].

Conclusions

In this paper we start with reviewing Lidsey's work on connection between Ermakov-Pinney equation with cosmology schemes, then we also review his further work which attempts to establish the connection with BEC experiments in lab and cosmology setting.

Nonetheless, our additional note in this paper is pertaining to the use of similar Ermakov equation in Newtonian Universe with vortex, which indicates early universe with rotation can be modeled using Ermakov equation instead of trying to modify Friedmann equation for *rotating metric*.

In retrospect, noting similarity between EP equation in Lidsey's work and ours; it seems to indicate that it is possible to consider a turbulence Universe in terms of BEC experiments too. (It may be worth noting here, that superfluid vortex dynamics may be modelled in classical turbulence too, but it is beyond the scope of this paper).

That is why, we call this approach: "*one note samba*" approach to cosmology. Further investigation and experiment are recommended in this direction.

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