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Conformally Static Spherically Symmetric Sphere with String Monopole and Cosmological Constant in Higher Dimension

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Abstract

In this paper, we have obtained static spherically symmetric solution of Einstein field equations in higher dimension with string monopole admitting one parameter group of conformal motion. For this, we have assumed the cosmological constant Λ to be a space-like variable scalar, viz., $\Lambda = \Lambda(r)$. The solutions are matched to the exterior solution of the higher dimensional Reissner-Nordstrom metric on the boundary $r = r_0$and discussed the result.

Keywords: Static, spherically symmetric, conformal sphere, cosmological constant, monopole, string.

1. Introduction

In recent year, the study of higher dimensions play an important role in super-string and Yang-Mills gravity (Schwarz 1985, Weinberg 1986)which demand more than usual 4-dimensional space-time. Kaluza (1921) and Klein (1926) made a remarkable theoretical description of gravity in fifth dimension. They realized that a unified theory of gravity and electromagnetism could exist in a world with 4+1 space-time dimensions. The fifth dimension could be curled up on a circle of radius R so small that nobody had observed it. This idea is particularly important in the field of cosmology to know the exact physical situation at very early stages of the formation of universe.

In the early stages of its evolution, the different types of topological objects may have been formed during phase transitions such as domain walls, cosmic string and monopoles (Kibble 1976). The basic idea is that these topological defects appeared due to breakdown of local or global gauge symmetries. This symmetry breaking is described within the particle physics in terms of the Higgs-Kibble mechanism. Suppose that we have the Higgs field $\varphi^a (a = 1,2, \ldots\ldots N)$whose potentials is

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$$V(\varphi) = \frac{\lambda}{4}(\varphi^2 - \eta_0^2\,)^2, \qquad \varphi^2 = \varphi^a\varphi^a,$$

where η_0 is the vacuum value and λ a coupling constant. This model gives rise to different types of topological models such as domain walls for N = 1, cosmic string for N = 2 and monopoles for N = 3. Farook Rahaman (2003) has studied the globle monopoles in higher dimensional space-time in Lyra geometry.

In this paper, we attempt to study the static spherically symmetric solution of Einstein field equations in higher dimension with string monopole which admit a one parameter group of conformal motion generated by a vector field ξ^α(K. Yano 1957) such that

$$\mathcal{L}_\xi g_{\alpha\beta} = 2\psi g_{\alpha\beta}\ ,$$

where the left hand side is the Lie derivative of the metric tensor with respect to the vector field ξ, and ψ is the conformal factor. We assumed the cosmological constant Λ to be a space-like variable scalar, viz., $\Lambda = \Lambda(r)$. In cosmology the dependence of Λ on time has been found to be of vital role playing now, its dependence on space coordinates is equally important for astrophysical problems (Saibal Ray *et al*. 2008, Chen and Wu 1990, Naralikar *et al*. 1991, Ray and Ray 1993, Tiwari and Ray 1996). The solutions are matched to the exterior solution of the higher dimensional Reissner-Nordstrom metric on the boundary $r = r_0$.

2. Einstein Field Equations for Global Monopoles in Higher Dimension

The higher dimensional static spherically symmetric space-time is taken as

$$ds^2 = e^{\nu(r)}dt^2 - e^{\mu(r)}dr^2 - r^2 d\chi^2 \qquad 1$$

with $d\chi^2 = d\theta_1^2 + sin^2\theta_1 d\theta_2^2\ + sin^2\theta_1 sin^2\theta_2 d\theta_3^2 +\ \dots +\ sin^2\theta_1 \dots sin^2\theta_{n-1} d\theta_n^2$

where the metric potentials ν and μ are the functions of radial coordinate r only.

Now, the Einstein field equations for the case of cosmological constant and string coupled with monopoles are

$$R_j^i - \frac{1}{2}R\delta_j^i + \Lambda\delta_j^i = -[\,T_j^{i\,(s)} + T_j^{i\,(m)}]\ , \qquad 2$$

where $T_j^{i\,(s)}$ and $T_j^{i\,(m)}$ are respectively, the energy-momentum tensor components for string and global monopoles, and Λ a cosmological constant.

The energy-momentum tensor components for string, given by Letelier (1979, 1983), is

$$T_j^{i\,(s)} = \rho u^i u_j - \rho_s x^i x_j, \qquad 3$$

where ρ is the rest energy density for the cloud of string with particle attached to them and ρ_s is the tension density of the string. Hence

$$\rho_s = \rho - \rho_p, \qquad 4$$

where ρ_p is the particle energy density.

Here $u^i = e^{-\frac{\nu}{2}}\delta_0^i$ is the (n+2) velocity and $x^i = e^{-\frac{\mu}{2}}\delta_1^i$ is the unit space-like vector in the radial direction which represent the string direction.

Now, a global monopole is a heavy object formed in the phase transition of a system composed by a self-coupling triplet scalar field φ^a $(a = 1,2,3)$(Bariolla and Vilenkin 1989, Banerji *et al*. 1996, Bazerra de Mello 2001 and FarookRahaman 2003).

Take the Lagrangian as

$$L = \frac{1}{2} g^{\alpha\beta}(\partial_\alpha \varphi^a)(\partial_\beta \varphi^a) - \frac{\lambda}{4}(\varphi^2 - \eta_0^2)^2. \qquad 5$$

The field configuration describing a monopole is $\varphi^a = \eta_0 f(r)\frac{x^a}{r}$, where $x^a x^a = r^2$.

The energy momentum tensor can be written by Banerji*et al*. (1996) as

$$T_{ab} = 2\frac{\partial L}{\partial g^{ab}} - L g_{ab},$$

$$T_t^t = -\eta_0{}^2\left[\frac{f^2}{r^2} + \frac{f'^2}{2e^\mu} + \frac{\lambda}{4}\eta_0{}^2(f^2-1)^2\right],$$

$$T_r^r = -\eta_0{}^2\left[\frac{f^2}{r^2} + \frac{f'^2}{2e^\mu} + \frac{\lambda}{4}\eta_0{}^2(f^2-1)^2\right],$$

$$T_{\theta_1}^{\theta_1} = T_{\theta_2}^{\theta_2} = \cdots = T_{\theta_{n-1}}^{\theta_{n-1}} = -\eta_0{}^2\left[\frac{f'^2}{2e^\mu} + \frac{\lambda}{4}\eta_0{}^2(f^2-1)^2\right],$$

where prime denotes the differentiation with respect to r. Outside the monopole core $f \sim 1$. With this result, the energy momentum tensor with (3) lead to

$$T_t^t = \rho + \frac{\eta_0{}^2}{r^2}, T_r^r = \rho_s + \frac{\eta_0{}^2}{r^2}, T_{\theta_1}^{\theta_1} = T_{\theta_2}^{\theta_2} = \cdots = T_{\theta_{n-1}}^{\theta_{n-1}} = 0\,. \qquad 6$$

The Einstein field equations (2) with (6) for the metric (1) lead to

$$e^{-\mu}\left[\frac{n\mu'}{2r} - \frac{n(n-1)}{2r^2}\right] + \frac{n(n-1)}{2r^2} - \Lambda = \rho + \frac{\eta_0{}^2}{r^2}\,, \qquad 7$$

$$e^{-\mu}\left[\frac{n\nu\prime}{2r}-\frac{n(n-1)}{2r^2}\right]+\frac{n(n-1)}{2r^2}-\Lambda=\rho_s+\frac{{\eta_0}^2}{r^2}\,, \qquad 8$$

$$e^{-\mu}\left[\frac{\nu''}{2}+\frac{{\nu'}^2}{4}-\frac{\nu'\mu'}{4}+\frac{(n-1)(\nu'-\mu')}{2r}+\frac{(n-1)(n-2)}{2r^2}\right]+\Lambda=0\,. \qquad 9$$

3. Solutions of the Field Equations

Now, we assume that the higher dimensional space-time admits a one-parameter group of conformal motions i.e.

$$\mathcal{L}_\xi g_{ik}=\xi_{i;k}+\xi_{k;i}=\psi g_{ik}\,. \qquad 10$$

where $\mathcal{L}_\xi$ is the Lie derivative of the metric tensor with respect to the vector field ξ, and ψ is the conformal factor which is the function of r . Yavuz and Yilmaz (1997), Yilmaz *et al*. (1999) ,Baysal *et al*. (2002) and Anirudh Pradhan*et al*. (2007) have considered the conformal motion in string cosmology.

By virtue of spherical symmetry, the equations (1) and (10) lead to the following expressions

$$\xi^1\nu'=\psi\,, \qquad 11$$

$$\xi^1=\frac{\psi\, r}{2}\,, \qquad 12$$

$$\xi^0=C=\text{constant} \qquad 13$$

and $$2\frac{\partial\xi^1}{\partial r}+\mu'\xi^1=\psi\,. \qquad 14$$

From equations (11) to (14), we get

$$e^\nu=A^2r^2\,, \qquad 15$$

$$e^\mu=\left(\frac{B}{\psi}\right)^2\,, \qquad 16$$

and$$\xi^\alpha=C\delta_0^\alpha+\left(\frac{\psi\, r}{2}\right)\delta_1^\alpha\,, \qquad 17$$

where A and B are constants of integration.

Now, substituting equations (15) and (16) in equations (7) – (9), we get

$$\rho+\frac{{\eta_0}^2}{r^2}+\Lambda=\frac{n(n-1)}{2r^2}\left[1-\frac{\Psi^2}{B^2}\right]-\frac{n\psi\psi\prime}{B^2r}\,. \qquad 18$$

$$\rho_s + \frac{{\eta_0}^2}{r^2} + \Lambda = -\frac{n(n+1)}{2}\frac{\Psi^2}{B^2 r^2} + \frac{n(n-1)}{2r^2} . \qquad 19$$

$$\Lambda = -\frac{n\Psi\Psi\prime}{B^2 r} - \frac{n(n-1)\Psi^2}{2B^2 r^2} + \frac{(n-1)(n-2)}{2r^2} . \qquad 20$$

Using (20) in (18) and (19), we get

$$\rho + \frac{{\eta_0}^2}{r^2} = \frac{n-1}{r^2} , \qquad 21$$

and $$\rho_s + \frac{{\eta_0}^2}{r^2} = \frac{n\psi\psi\prime}{B^2 r} - \frac{n\psi^2}{B^2 r^2} + \frac{n-1}{r^2} . \qquad 22$$

From equations (21) and (22), we get

$$\rho_p = \rho - \rho_s = \frac{n\psi}{B^2 r}\left[\frac{\psi}{r} - \psi'\right] . \qquad 23$$

Using $\rho_p = 0$ in equation (23), after simplification, we get

$$\psi = Dr ,$$

where D is a constant of integration.

Using equations (15) and (16), the higher dimensional static spherically symmetric space-time geometry of string coupled with global monopole becomes

$$ds^2 = A^2 r^2 dt^2 - \frac{B^2}{\psi^2} dr^2 - r^2 d\chi^2 . \qquad 24$$

Using $\psi = Dr$, then

$$\rho = \rho_s = \frac{1}{r^2}[n - (\eta_0^2 + 1)] \text{ and}$$

$$\Lambda = \frac{(n-1)(n-2)}{2r^2} - \frac{n(n+1)D^2}{2B^2} .$$

4. Matching with Reissner Nordstrom Metric

The solution of Einstein field equations for $r > r_0$ is given by the higher dimensional Reissner Nordstrom Metric as

$$ds^2 = \left[1 - \frac{2M}{r^{n-1}} - \frac{\Lambda n(n-1) r^{2(n-1)}}{6}\right] dt^2 - \left[1 - \frac{2M}{r^{n-1}} - \frac{\Lambda n(n-1) r^{2(n-1)}}{6}\right]^{-1} dr^2 - r^2 d\chi^2 \qquad 25$$

To match the space-time geometry (24) with the Reissner Nordstrom Metric (25) across the boundary $= r_0$, we require continuity of gravitational potential g_{ij} at $r = r_0$.

$$A^2 r_0^2 = (\frac{\psi}{B})^2 = \left[1 - \frac{2M}{r^{n-1}} - \frac{\Lambda n(n-1) r^{2(n-1)}}{6}\right]. \qquad 26$$

Using $\psi = D r_0$ in equation (26), after simplification we get

$$\frac{M}{r_0^{n-1}} = \frac{1}{2} - \frac{{r_0}^2}{2} - \frac{\Lambda n(n-1) r_0^{2(n-1)}}{12}.$$

This gives

$$M = \frac{r_0^{n-1}}{2} - \frac{r_0^{n+1}}{2} - \frac{\Lambda n(n-1) r_0^{3(n-1)}}{12}.$$

5. Conclusions

In this paper, we have studied the static spherically symmetric solution of Einstein field equations in higher dimension with string attached to monopole which admit a one parameter group of conformal motion generated by a vector field ξ^α . For this, we assumed the cosmological constant Λ to be a function of radial coordinate r only. We have observed that e^ν and e^μ are both positive, continuous and non-singular for $r < r_0$. In this case, we have matched our solutions with the Reissner Nordstrom Metric at $r = r_0$ and obtained string solutions with monopole.

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