Article

Non-local Superpotential for the Radiative Part of the Liénard-Wiechert's Electromagnetic Field

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Abstract

For a classical particle charged in arbitrary motion, we give an elementary deduction of a non-local generator for the radiative part of the electromagnetic field that it produces.

Keywords: Liénard-Wiechert field, Maxwell tensor.

1. Introduction

We shall employ the conventions and quantities explained in [1-6]. The radiative part of the Maxwell tensor for the Liénard-Wiechert field [7-9] is given by [3, 10]:

$$T_{bc} = q^2\, w^{-4}\, (a^2 - w^{-2}\, W^2)\, k_b\, k_c, \qquad W = -a^r k_r\,, \tag{1}$$

and we search a potential $K_{cjb} = -K_{jcb}$ such that:

$$T_{bc} = K_c{}^j{}_{b,j}, \tag{2}$$

thus the conservation law $T^{bc}{}_{,j} = 0$ is an identity. The generator K_{abc} must be non-local because is associated with the electromagnetic radiation emitted by the particle, that is, such potential shall depend of integrals over the past history of the charge.

The corresponding Faraday tensor $F_{bc} = -F_{cb}$ verifies the Maxwell equations:

$$F_{bc,r} + F_{cr,b} + F_{rb,c} = 0, \qquad F^{bc}{}_{,c} = 0, \tag{3}$$

such that:

$$F_{bc}\, k^c = q\, w^{-2}\, k_b\,, \qquad \tau_{,j} = -w^{-1}\, k_j\,, \qquad k^r\, k_r = 0, \tag{4}$$

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with the participation of the proper time τ and the retarded distance w from a point field to C, the trajectory of the point charge q.
We shall use an orthonormal tetrad on C where $e_{(4)b} = v_b$ is the 4-velocity, $v^r v_r = -1$, and $e_{(\sigma)b}$, $\sigma = 1, 2, 3$ is a unitary space-like Fermi triad [11, 12] that evolves on C according to the transport law:

$$\frac{d}{d\tau}\, {e_{(\sigma)}}^b = a_{(\sigma)}\, v^b\,, \qquad a_{(\gamma)} = a^r\, e_{(\gamma)r}\,, \tag{5}$$

that is, $a_{(\sigma)}$ are the projections of the acceleration over the spatial triad; then we can prove the important property:

$$F^{bc}\, p_{(\sigma),c} = 0, \quad p_{(\sigma)} = p^r\, e_{(\sigma)r}\,, \quad p^r = w^{-1} k^r - v^r\,. \tag{6}$$

In Sec. 2 we employ (3, …, 6) to deduce the potential K_{abc} for the radiative part (1).

2. Generator for the radiative tensor of Liénard-Wiechert

From (1, 4):

$$T_{bc} = q\, w^{-2}\, {F_c}^j\, (a^2 - w^{-2}\, W^2)\, k_b\, k_j, \qquad W = -w\, a^r p_r = -a_{(\gamma)}\, p_{(\gamma)}\,,$$

$$= -q\, {F_c}^j\, \left(a^2 - a_{(\sigma)}\, p_{(\sigma)}\, a_{(\beta)}\, p_{(\beta)}\right)\left(v_b + p_{(\gamma)}\, e_{(\gamma)b}\right) \tau_{,j}\,, \tag{7}$$

then we observe that:

$${F_c}^j\, \tau_{,j}\, a^2\, v_b \overset{(3)}{=} ({F_c}^j \textstyle\int_0^\tau a^2\, v_b\, d\lambda)_{,j}\,, \qquad {F_c}^j\, \tau_{,j}\, a^2\, e_{(\gamma)b}\, p_{(\gamma)} \overset{(3,6)}{=} ({F_c}^j\, p_{(\gamma)} \textstyle\int_0^\tau a^2\, e_{(\gamma)b}\, d\lambda)_{,j}\,,$$

$${F_c}^j\, \tau_{,j}\, a_{(\sigma)}\, a_{(\beta)}\, v_b\, p_{(\sigma)}\, p_{(\beta)} \overset{(3,6)}{=} ({F_c}^j\, p_{(\sigma)}\, p_{(\beta)} \textstyle\int_0^t a_{(\sigma)}\, a_{(\beta)}\, v_b\, d\lambda)_{,j}\,, \quad \text{etc.} \tag{8}$$

thus (7) acquires the structure (2) with the non-local potential [4]:

$$K_{cjb} = qF_{cj}\Big[p_{(\sigma)}\Big(p_{(\beta)} \textstyle\int_0^\tau a_{(\sigma)}\, a_{(\beta)} v_b d\lambda + p_{(\beta)} p_{(\gamma)} \int_0^\tau a_{(\sigma)}\, a_{(\beta)} e_{(\gamma)b} d\lambda - \int_0^\tau a^2\, e_{(\sigma)b} d\lambda\Big) - \int_0^\tau a^2\, v_b d\lambda\Big],$$

(9)

which is zero in the absence of acceleration. The possible physical meaning of (9), is an open problem.

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