Review Article

A Review on Nuclear Binding Energy Connected with Strong Interaction

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Abstract

Nuclear binding energy can be addressed with a single energy coefficient assumed to be associated with strong and coulombic interactions.

Keywords: Beta stability line, semi-empirical mass formula, strong coupling constant.

1. Introduction

Understanding and estimating nuclear binding energy with 'strong interaction' [1] seems to be quite interesting and needs a serious review at basic level. In this context, so far we have developed many possible relations with 3 to 4 versions [2-6]. Our basic idea is that, nuclear binding energy can be addressed with a single energy coefficient assumed to be associated with strong interaction. According to semi empirical mass formula (SEMF) [7-9]: 1) There exist 5 different energy terms and 5 different energy coefficients. 2) Average binding energy per nucleon is approximately 8 MeV and maximum binding energy per nucleon is around 8.8 MeV. It may be noted that, SEMF is lagging in implementing the strong coupling constant in understanding nuclear binding energy scheme. Keeping this fact in view, in a unified approach, in this paper, we reviewed our views on nuclear binding energy with respect to beta stability line, SEMF and strong coupling constant, $(\alpha_s \cong 0.1186)$. We retain sections 2 and 3 for better presentation and clarity. Our proposal is much simple and much realistic than the new integrated model proposed by N. Ghahramany et al. [10,11,12]. Close to the beta stability line, if $k \cong (1/4\pi)^2 \cong 0.006333,$ for $Z \cong (40 \text{ to } 83),$ binding energy can be expressed with:

$$(B)_A \cong \left[A - \left(\frac{kAZ}{3}\right)\right] \times 9.5 \text{ MeV} \cong A\left[1 - \left(\frac{kZ}{3}\right)\right] \times 9.5 \text{ MeV} \quad (1)$$

See table -1.

Table-1: To fit the nuclear binding energy of Z = (40 to 83) close to the beta stability line

Proton	Mass number	Estimated	Estimated	Actual [10]	Error

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number	close to beta stability line	stable neutron number	binding energy(MeV)	or Reference [8] binding energy (MeV)	w.r.t actual or reference binding energy (MeV)
40	90	50	782.8	783.893	1.1
41	93	52	807.0	805.765	-1.3
42	95	53	822.5	821.625	-0.9
43	98	55	846.5	844.4	-2.1
44	100	56	861.8	861.928	0.2
45	103	58	885.5	884.163	-1.4
46	105	59	900.6	899.914	-0.7
47	108	61	924.2	922.2	-2.0
48	111	63	947.6	947.622	0.0
49	113	64	962.5	963.094	0.6
50	116	66	985.7	988.684	3.0
51	118	67	1000.3	1000.48	0.2
52	121	69	1023.3	1024.43	1.1
53	124	71	1046.2	1046.32	0.1
54	126	72	1060.5	1063.909	3.4
55	129	74	1083.2	1085.08	1.9
56	132	76	1105.8	1110.038	4.3
57	135	78	1128.2	1131	2.8
58	137	79	1142.1	1145.7	3.6
59	140	81	1164.3	1168.67	4.3
60	143	83	1186.4	1191.266	4.8
61	146	85	1208.4	1209.52	1.1
62	148	86	1222.0	1225.392	3.4
63	151	88	1243.7	1244.141	0.4
64	154	90	1265.3	1266.627	1.3
65	157	92	1286.8	1287.38	0.5
66	160	94	1308.2	1309.455	1.2
67	162	95	1321.3	1321.18	-0.1
68	165	97	1342.5	1343.08	0.6
69	168	99	1363.5	1363.31	-0.2
70	171	101	1384.4	1384.744	0.3
71	174	103	1405.2	1404.44	-0.8
72	177	105	1425.9	1425.185	-0.7
73	180	107	1446.5	1444.663	-1.8
74	183	109	1466.9	1465.525	-1.4
75	186	111	1487.2	1484.63	-2.6
76	189	113	1507.4	1505.007	-2.4
77	192	115	1527.5	1523.81	-3.7
78	195	117	1547.5	1545.682	-1.8
79	198	119	1567.3	1564.94	-2.4
80	201	121	1587.0	1587.411	0.4
81	204	123	1606.6	1606.87	0.3
82	207	125	1626.1	1629.063	3.0
83	210	127	1645.5	1643.94	-1.5

2. About the Liquid drop model and the semi empirical mass formula

According to liquid drop model,

1) Atomic nucleus can be considered as a drop of incompressible fluid.
2) Nuclear fluid is made of protons and neutrons, which are held together by the strong nuclear force.

Mathematical analysis of the theory delivers an equation which attempts to predict the binding energy of a nucleus in terms of the numbers of protons and neutrons it contains. This equation has five terms on its right hand side. These correspond to the cohesive binding of all the nucleons by the strong nuclear force, the electrostatic mutual repulsion of the protons, a surface energy term, an asymmetry term (derivable from the protons and neutrons occupying independent quantum momentum states) and a pairing term (partly derivable from the protons and neutrons occupying independent quantum spin states). The coefficients are calculated by fitting to experimentally measured masses of nuclei. Their values can vary depending on how they are fitted to the data. In the following formulae, let A be the total number of nucleons, Z *the* number of protons and N the number of neutrons. According to semi-empirical mass formula,

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} \pm \frac{a_p}{\sqrt{A}} \tag{2}$$

Here a_v = volume energy coefficient, a_s is the surface energy coefficient, a_c is the coulomb energy coefficient, a_a is the asymmetry energy coefficient and a_p is the pairing energy coefficient. If we consider the sum of the volume energy, surface energy, coulomb energy, asymmetry energy and pairing energy, then the picture of a nucleus as a drop of incompressible liquid roughly accounts for the observed variation of binding energy of the nucleus. See table-2 for various semi empirical mass formula energy coefficients [9, 13].

Table 2: SEMF binding energy coefficients

Reference	a_v MeV	a_s MeV	a_c MeV	a_a MeV	a_p MeV
[3]	15.78	18.34	0.71	23.21	12.0
[3]	15.258	16.26	0.689	22.20	10.08
[13]	16.32	19.59	0.769	23.94	11.97

3. Estimation of stable mass number with proton number

According to SEMF, by maximizing $B(A,Z)$ with respect to Z, it is possible to find the number of protons Z of the stable nucleus of atomic weight A as,

$$Z \approx \frac{A}{2+\left[\left(a_c/2a_a\right)A^{2/3}\right]} \quad (3)$$

This is roughly $A/2$ for light nuclei, but for heavy nuclei there is an even better agreement with nature. By substituting the above value of Z back into B one obtains the binding energy as a function of the atomic weight, $B(A)$. Maximizing $B(A)/A$ with respect to A gives the nucleus which is most strongly bound or most stable. Another interesting stability relation is:

$$(A-2Z) \approx \frac{0.4A^2}{A+200} \quad (4)$$

In this context, we would like to suggest that, independent of SEMF concepts, nuclear beta stability line can be understood with neutron, proton and electron rest masses. Numerically, it is possible to show that,

$$\begin{aligned} &\exp\left(\frac{\left(m_n-m_p\right)c^2}{m_ec^2}\right) \cong 12.5659102 \cong 4\pi \\ &\rightarrow \left(\frac{\left(m_n-m_p\right)c^2}{m_ec^2}\right) \cong \ln\left(4\pi\right) \cong 2.53102425 \end{aligned} \quad (5)$$

where, $m_nc^2 \cong 939.565413$ MeV, $m_pc^2 \cong 938.272081$ MeV and $m_ec^2 \cong 0.5109989461$ MeV. Based on this observation and without considering the binding energy coefficients, beta stability line can be understood with the following empirical relations.

$$\left.\begin{aligned} &\text{Let, } k \cong \left(\frac{1}{4\pi}\right)^2 \cong 0.006333 \\ &A_s \cong 2Z + 0.006333Z^2 \cong 2Z + kZ^2 \\ &N_s \cong Z + 0.006333Z^2 \cong Z + kZ^2 \\ &\quad \rightarrow A_s - 2Z \cong 0.006333Z^2 \cong kZ^2 \\ &\quad \Rightarrow \quad \frac{Z}{\sqrt{A_s-2Z}} \cong \sqrt{\frac{1}{k}} \cong 4\pi \end{aligned}\right\} \quad (6)$$

These relations can be compared with the computational relation pertaining to isotonic shift and drip lines proposed in reference [9], $N_s = 0.968051Z + 0.00658803Z^2$. With even-odd corrections much better correlations can be observed. For light and medium atomic nuclides, there is some mismatch. It can be attributed to shell structure and needs for further study. See table-3 for fitting the stable nucleon number with its corresponding proton number. Using the proposed number k, nuclear binding energy can also be estimated. In this direction, we have developed various relations and we are trying to integrate them to get accuracy with possible background physics.

In table 3, one can see the remarkable fitting of estimated and actual stable isotopes. It is also true that, in between $Z = 51 \text{ and } 60$, there is some continuous deviation in the estimated data and needs fine tuning. Super heavy stable elements can also be predicted with this relation. Results obtained with the above relation can be compared with SEMF stability relations (3) and (4). It is very interesting to note that, table 3 can be reproduced with a very simple relation associated with proton number and strong coupling constant. See section 6.

Table-3 : To fit the stable mass numbers

Proton number	Estimated stable mass number	Observed stable mass number(s)		Proton number	Estimated stable mass number	Observed stable mass number(s)
11	22.77	23		51	118.47	121
21	44.79	45		53	123.78	127
23	49.34	50,51		55	129.16	133
25	53.96	55		57	134.57	138,139
27	58.62	59		59	140.0	141
29	63.33	63,65		63	151.13	151
31	68.08	69,71		65	156.75	159
33	72.90	75		67	162.43	165
35	77.76	79		69	168.15	169
37	82.67	85,87		71	173.92	175,176
39	87.63	89		73	179.75	180,181
41	92.64	93		75	185.6	185,187
43	97.71	--		79	197.52	197
45	102.82	103		81	203.55	203,205
47	107.99	107,109		83	209.62	209
49	113.2	113/115		92	237.60	234,235,238

4. Proposed new concepts and semi empirical relations of nuclear binding energy

We would like to suggest that:

1) Reciprocal of the strong coupling constant can be considered as an index of strength of nuclear binding energy.
2) $\left[\left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0}\right] \cong (9.6 \text{ to } 9.7)$ MeV can be considered as a characteristic nuclear binding energy potential pertaining to strong interaction.

3) $\left[\left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0}\right]-\left[\left(\frac{3}{5}\right)\frac{e^2}{4\pi\varepsilon_0 R_0}\right]\cong(8.9\text{ to }9.0)$ MeV can be considered as an effective nuclear binding energy potential pertaining to strong interaction and coulombic repulsion.
4) (α_s) and $\exp\left(\frac{m_n-m_p}{m_e}\right)\cong 4\pi$ seem to play a crucial role in understanding the beta stability line and nuclear binding energy.
5) Asymmetry term and ratio of mass number to proton number seem to play a key role in reducing binding energy.
6) Stable isotope of beta stability line plays an important role in estimating the binding energy of other stable and unstable isotopes of Z.
7) Ratio of neutron number to stable neutron number seems to play a crucial role in estimating the binding energy of corresponding neutron number.

4.1 Characteristic nuclear binding energy potential B_0

Characteristic nuclear binding energy potential can be addressed in the following way.

With reference to semi empirical mass formula, coulombic energy constant is given by,

$$\begin{aligned} a_c &\cong \left(\frac{3}{5}\right)\frac{e^2}{4\pi\varepsilon_0 R_0}\cong 0.71\text{ MeV} \\ \text{where } R_0 &\cong (1.24\text{ to }1.25)\text{ fm} \end{aligned} \tag{7}$$

Based on this relation and with reference to the strong coupling constant, we assume that,

$$B_0 \cong \left[\left(\frac{1}{\alpha_s}\right)\frac{e^2}{4\pi\varepsilon_0 R_0}\right]-\left[\left(\frac{3}{5}\right)\frac{e^2}{4\pi\varepsilon_0 R_0}\right]\cong(9.71-0.71)\cong 9.0\text{ MeV} \tag{8}$$

It is possible to interpret that, characteristic strong interaction nuclear binding energy potential is around 9.71 MeV and after counteracting the coulombic interaction, effective binding energy potential seems to stand at around (8.9 to 9.0) MeV.

4.2 New relations for nuclear binding energy

Case 1 : Starting from Z=26, close to the line of stability,

$$B_{A_s} \cong \left[A_s - \left(\frac{(A_s - 2Z)^2}{2A_s} \right) - \left(\frac{(A_s - 2Z)^2}{Z} \right) \right] B_0$$
$$\cong \left[A_s - \left\{ \left(\frac{(A_s - 2Z)^2}{A_s} \right) \left(\frac{A_s}{Z} + \frac{1}{2} \right) \right\} \right] \times 9.0 \text{ MeV} \quad (9)$$
$$\text{where, } A_s \cong 2Z + (Z^2 k) \cong 2Z + (Z/4\pi)^2$$

See table 4 for estimated nuclear binding energy for Z=26 to 100.

Note: $Z = 83$ is estimated to be stable at $A_s = 210$. With even-odd correction scheme, if it is granted that, $A_s = 209$, then, from relation (9), estimated binding energy of ${}_{83}Pb^{209}$ is 1640.70 MeV and its actual binding energy is 1640.23 MeV.

Table-4. Estimated nuclear binding energy of Z = (26 to 100) close to beta stability line

Proton number	Estimated stable mass number	Estimated stable neutron number	Estimated binding energy(MeV) Relation (9)	Actual [10] or Reference [9] binding energy (MeV)	Error w.r.t actual or reference binding energy (MeV)
26	56	30	497.2	492.258	-4.94
27	59	32	520.8	517.313	-3.49
28	61	33	539.1	534.666	-4.43
29	63	34	557.5	551.385	-6.12
30	66	36	580.7	578.136	-2.56
31	68	37	599.2	590.61	-8.59
32	70	38	617.6	610.521	-7.08
33	73	40	640.6	634.34	-6.26
34	75	41	659.1	651.02	-8.08
35	78	43	681.9	676.11	-5.79
36	80	44	700.4	695.434	-4.97
37	83	46	722.9	720.46	-2.44
38	85	47	741.5	737.85	-3.65
39	88	49	763.8	763.88	0.08
40	90	50	782.5	783.893	1.39
41	93	52	804.6	805.765	1.16
42	95	53	823.3	821.625	-1.67
43	98	55	845.2	844.40	-0.80
44	100	56	864.1	861.928	-2.17
45	103	58	885.8	884.163	-1.64
46	105	59	904.7	899.914	-4.79
47	108	61	926.3	922.2	-4.10
48	111	63	947.7	947.622	-0.08
49	113	64	966.7	963.094	-3.61
50	116	66	988.0	988.684	0.68

51	118	67	1007.1	1000.48	-6.62
52	121	69	1028.2	1024.43	-3.77
53	124	71	1049.2	1046.32	-2.88
54	126	72	1068.4	1063.909	-4.49
55	129	74	1089.3	1085.08	-4.22
56	132	76	1110.1	1110.038	-0.06
57	135	78	1130.7	1131.00	0.30
58	137	79	1150.1	1145.7	-4.40
59	140	81	1170.6	1168.67	-1.93
60	143	83	1191.0	1191.266	0.27
61	146	85	1211.3	1209.52	-1.78
62	148	86	1230.9	1225.392	-5.51
63	151	88	1251.1	1244.141	-6.96
64	154	90	1271.2	1266.627	-4.57
65	157	92	1291.2	1287.38	-3.82
66	160	94	1311.0	1309.455	-1.55
67	162	95	1330.9	1321.18	-9.72
68	165	97	1350.8	1343.08	-7.72
69	168	99	1370.5	1363.31	-7.19
70	171	101	1390.2	1384.744	-5.46
71	174	103	1409.7	1404.44	-5.26
72	177	105	1429.2	1425.185	-4.02
73	180	107	1448.6	1444.663	-3.94
74	183	109	1467.9	1465.525	-2.38
75	186	111	1487.1	1484.63	-2.47
76	189	113	1506.3	1505.007	-1.29
77	192	115	1525.4	1523.81	-1.59
78	195	117	1544.4	1545.682	1.28
79	198	119	1563.4	1564.94	1.54
80	201	121	1582.3	1587.411	5.11
81	204	123	1601.1	1606.87	5.77
82	207	125	1619.9	1629.063	9.16
83	210	127	1638.6	1643.94	5.34
84	213	129	1657.3	1659.72	2.42
85	216	131	1675.9	1673.42	-2.48
86	219	133	1694.4	1690.59	-3.81
87	222	135	1713.0	1706.49	-6.51
88	225	137	1731.4	1724.18	-7.22
89	228	139	1749.8	1740.67	-9.13
90	231	141	1768.2	1759.14	-9.06
91	234	143	1786.6	1776.08	-10.52
92	238	146	1801.6	1801.69	0.09
93	241	148	1819.8	1817.31	-2.49
94	244	150	1837.9	1835.45	-2.45
95	247	152	1856.0	1851.73	-4.27
96	250	154	1874.1	1868.97	-5.13
97	254	157	1888.2	1888.79	0.59
98	257	159	1906.1	1906.19	0.09
99	260	161	1924.0	1922.20	-1.80
100	263	163	1941.9	1939.52	-2.38

Case 2 : For light atoms, starting from Z = (3 to 30), approximately, nuclear binding energy can be expressed by the following relation.

$$B_{A_s} \cong \left[A_s - \left(\frac{A_s}{Z} + \frac{1}{2} \right) \right] \times 9.2 \text{ MeV} \tag{10}$$

This relation seems to be applicable for $A \approx (A_s + 1)$ to $(A_s + 5)$ also. See table 5.

Table-5. Estimated nuclear binding energy of Z = (3 to 30) close to stability line

Proton number	Actual stable mass number	Estimated binding energy(MeV) Relation (10)	Actual binding energy [10] (MeV)	Error w.r.t actual (MeV)
3	6	32.2	31.994	-0.2
4	9	57.5	58.165	0.7
5	10	69.0	64.751	-4.2
6	12	87.4	92.162	4.8
7	14	105.8	104.659	-1.1
8	16	124.2	127.619	3.4
9	19	150.8	147.801	-3.0
10	21	169.3	167.406	-1.9
11	23	187.8	186.564	-1.2
12	25	206.2	205.588	-0.6
13	27	224.7	224.952	0.3
14	29	243.1	245.011	1.9
15	31	261.6	262.917	1.3
16	34	288.7	291.839	3.2
17	35	298.5	298.21	-0.2
18	38	325.6	327.343	1.8
19	40	344.0	343.811	-0.2
20	43	371.2	369.829	-1.4
21	45	389.7	387.848	-1.8
22	47	408.1	407.073	-1.1
23	50	435.4	434.794	-0.6
24	52	453.9	456.349	2.5
25	55	481.2	482.075	0.9
26	56	490.8	492.258	1.5
27	59	518.1	517.313	-0.8
28	61	536.6	534.666	-1.9
29	63	555.0	551.385	-3.6
30	66	582.4	578.136	-4.2

Case 3: For case-1, below and above the stable mass numbers, approximately, binding energy can be estimated by the following simple relation.

$$B_A \cong \left(\frac{A-Z}{A_s-Z}\right)^p \left(B_{A_s}\right)$$
$$\text{where } \left\{ \text{if } \left(A < A_s\right),\ p \cong \frac{2}{3};\ \text{if } \left(A > A_s\right),\ p \cong \frac{1}{2} \right. \tag{11}$$

For example,

1) $Z = 26$ is estimated to be stable at $A_s = 56$. With relation (9), its estimated binding energy is 497.2 MeV and with relation (10), its estimated binding energy is 490.8 MeV. Actual binding energy is 492.258 MeV. $Z = 26$ is also stable at $A_s = 58$. With reference to the actual binding energy of $_{26}Fe^{56}$, from relation (11), estimated binding energy of $_{26}Fe^{58}$ is 508.402 MeV and its actual binding energy is 509.949 MeV.
2) $Z = 33$ is estimated to be stable at $A_s = 73$. With relation (9) its estimated binding energy is 640.6 MeV. Actually, it is stable at $A_s = 75$. From relation (11), estimated binding energy of $_{33}As^{75}$ is 656.42 MeV and its actual binding energy is 652.564 MeV.
3) $Z = 53$ is estimated to be stable at $A_s = 124$. With relation (9) its estimated binding energy is 1049.22 MeV. Actually, $Z = 53$ is stable at $A_s = 127$. From relation (11), estimated binding energy of $_{53}I^{127}$ is 1071.16 MeV and its actual binding energy is 1072.57 MeV.
4) $Z = 67$ is estimated to be stable at $A_s = 162$. With relation (9) its estimated binding energy is 1330.9 MeV. Actually, $Z = 67$ is stable at $A_s = 165$. From relation (11), estimated binding energy of $_{67}Ho^{165}$ is 1351.75 MeV and its actual binding energy is 1344.256 MeV.

See the following table 6 for estimating the isotopic binding energy of Z=50.

Table-6. Estimated isotopic nuclear binding energy of Z=50

Proton number	Mass number	Neutron number	Estimated binding energy (MeV) Relations (9) and (11)	Reference binding energy [9] MeV	Error w.r.t Reference binding energy (MeV)
50	106	56	885.5	894.58	9.08
50	107	57	896.0	903.51	7.51
50	108	58	906.5	914.94	8.44
50	109	59	916.8	923.53	6.73
50	110	60	927.2	934.70	7.5
50	111	61	937.4	942.89	5.49
50	112	62	947.7	953.5	5.8
50	113	63	957.8	961.1	3.3
50	114	64	967.9	971.41	3.51
50	115	65	978.0	978.73	0.73
50	**116**	**66**	**988.0**	988.47	0.47
50	117	67	995.5	995.44	-0.06

50	118	68	1002.9	1004.74	1.84
50	119	69	1010.2	1011.26	1.06
50	120	70	1017.5	1020.32	2.82
50	121	71	1024.7	1026.76	2.06
50	122	72	1031.9	1035.51	3.61
50	123	73	1039.1	1041.54	2.44
50	124	74	1046.2	1050.08	3.88
50	125	75	1053.2	1055.84	2.64
50	126	76	1060.2	1064.06	3.86

5. Alternative simplified approach for estimating nuclear binding energy

We noticed that, close to the line of beta stability,

$$\frac{k^2 A_s^2 \left(A_s - Z\right)}{\sqrt{Z}} \approx \frac{\left(A_s - 2Z\right)^2}{Z} \tag{12}$$

$$\left.\begin{array}{l} \text{A) } k^2 A_s \left(A_s - Z\right)\sqrt{Z} \approx \dfrac{\left(A_s - 2Z\right)^2}{A_s} \\ \text{B) } \dfrac{A_s^{1/2}\left(A_s - Z\right)^{1/4} Z^{1/8}}{\sqrt{A_s - 2Z}} \approx \dfrac{1}{\sqrt{k}} \approx 4\pi \\ \text{C) } A_s^{4/7}\left(A_s - Z\right)^{2/7} \approx Z \end{array}\right\} \tag{13}$$

Based on these observations, we are working on $Z \geq 26$ for integrating the proposed relations (9) and (11) into one relation as [5]:

$$B_A \cong \left\{ A - \left[kN\left(\frac{kAA_s}{\sqrt{Z}} + 1 \right) \right] \right\} \times 8.9 \text{ MeV} \tag{14}$$

This energy unit, $B_0 \cong 8.9$ MeV can be understood in the following ways.

$$\underline{\text{Significance-1:}} \; \frac{B_0}{a_c} \cong 4\pi \cong \sqrt{\frac{1}{k}}$$
$$\rightarrow B_0 \cong \sqrt{\frac{1}{k}}\left(a_c\right) \cong 4\pi a_c \cong 8.922 \text{ MeV} \tag{15}$$

$$\underline{\text{Significance-2:}} \; \frac{B_0^2}{\left(m_p c^2\right) a_c} \cong \alpha_s \cong 0.1186$$
$$\rightarrow B_0 \cong \sqrt{\alpha_s \left(m_p c^2\right)\left(a_c\right)} \cong 8.89 \text{ MeV} \tag{16}$$

where $\alpha_s =$ Strong coupling constant [1].

$$\underline{\text{Significance-3:}}\ \frac{B_0}{m_p c^2} \cong \sqrt{k}\alpha_s \cong \frac{\alpha_s}{4\pi}$$
$$\rightarrow B_0 \cong \sqrt{k}\alpha_s\left(m_p c^2\right) \cong 8.856 \text{ MeV} \quad (17)$$

These relations are independent of the nuclear charge radius, R_0. Based on these relations, it is also possible to show that,

$$a_c \cong k\alpha_s m_p c^2 \cong 0.705 \text{ MeV} \quad (18)$$

With further study, other hidden relations can also be developed. See the following table 7 for the binding energy of natural isotopes of Z = 30, 40, 50,60,70,80 and 92.

Table-7. Estimated isotopic nuclear binding energy of Z = 30, 40, 50,60,70,80 and 92

Proton number	Estimated stable mass number	Actual stable mass number	Actual stable neutron number	Estimated binding energy MeV Relation(14)	Actual binding energy [10] MeV	Error w.r.t actual binding energy MeV
30	66	64	34	558.3	559.098	0.77
30	66	66	36	575.2	578.136	2.98
30	66	67	37	583.6	585.189	1.64
30	66	68	38	591.9	595.387	3.44
30	66	70	40	608.7	611.087	2.38
40	90	90	50	775.3	783.893	8.57
40	90	91	51	783.5	791.087	7.64
40	90	92	52	791.6	799.722	8.15
40	90	94	54	807.8	814.677	6.90
40	90	96	56	823.9	828.996	5.06
50	116	112	62	952.6	953.532	0.89
50	116	114	64	968.3	971.574	3.30
50	116	115	65	976.1	979.121	3.06
50	116	116	66	983.8	988.684	4.84
50	116	117	67	991.6	995.627	4.01
50	116	118	68	999.4	1004.955	5.57
50	116	119	69	1007.1	1011.438	4.31
50	116	120	70	1014.9	1020.546	5.68
50	116	122	72	1030.3	1035.53	5.23
50	116	124	74	1045.7	1049.963	4.27
60	143	142	82	1182.4	1185.142	2.69
60	143	143	83	1189.8	1191.266	1.46
60	143	144	84	1197.2	1199.083	1.93
60	143	145	85	1204.5	1204.838	0.35
60	143	146	86	1211.8	1212.403	0.59
60	143	148	88	1226.4	1225.028	-1.39
60	143	150	90	1241.0	1237.448	-3.52
70	171	168	98	1369.6	1362.793	-6.77

70	171	170	100	1383.3	1378.13	-5.21
70	171	171	101	1390.2	1384.744	-5.46
70	171	172	102	1397.1	1392.764	-4.29
70	171	173	103	1403.9	1399.131	-4.76
70	171	174	104	1410.7	1406.595	-4.12
70	171	176	106	1424.3	1419.283	-5.04
80	201	196	116	1555.5	1551.218	-4.27
80	201	198	118	1568.1	1566.489	-1.64
80	201	199	119	1574.4	1573.153	-1.28
80	201	200	120	1580.7	1581.181	0.46
80	201	201	121	1587.0	1587.411	0.42
80	201	202	122	1593.2	1595.165	1.93
80	201	204	124	1605.7	1608.652	2.96
92	238	234	142	1780.3	1778.567	-1.73
92	238	235	143	1785.8	1783.864	-1.93
92	238	238	146	1802.2	1801.69	-0.51

6. Understanding beta stability line with strong coupling constant

With trial- error method, we noticed that, close to the line of beta stability,

$$\text{for } Z \geq 9,\ A_s \cong \left(Z + \sqrt{\frac{1}{\alpha_s}} \right)^{\frac{6}{5}} \cong \left(Z + 2.904 \right)^{1.2} \tag{19}$$

$$\left. \begin{array}{l} \text{for } Z \geq 14,\ \left(A_s - 2Z \right) \cong \left(Z\beta + 1 \right)^2 - 4 \\ \text{where } \beta = \left(\dfrac{3}{5} \right) \alpha_s. \end{array} \right\} \tag{20}$$

Based on these relations, stable mass number can be estimated directly with proton number. These two relations can be compared with relation (6). Both relations can be applied for estimating the best possible range of stable isotopes of super heavy elements. See the following table -8. See figure-1 for data comparison. Blue curve indicates the mass numbers estimated with relation (6), red curve indicates the mass numbers estimated with relation (19) and blue curve indicates the mass numbers estimated with relation (20).

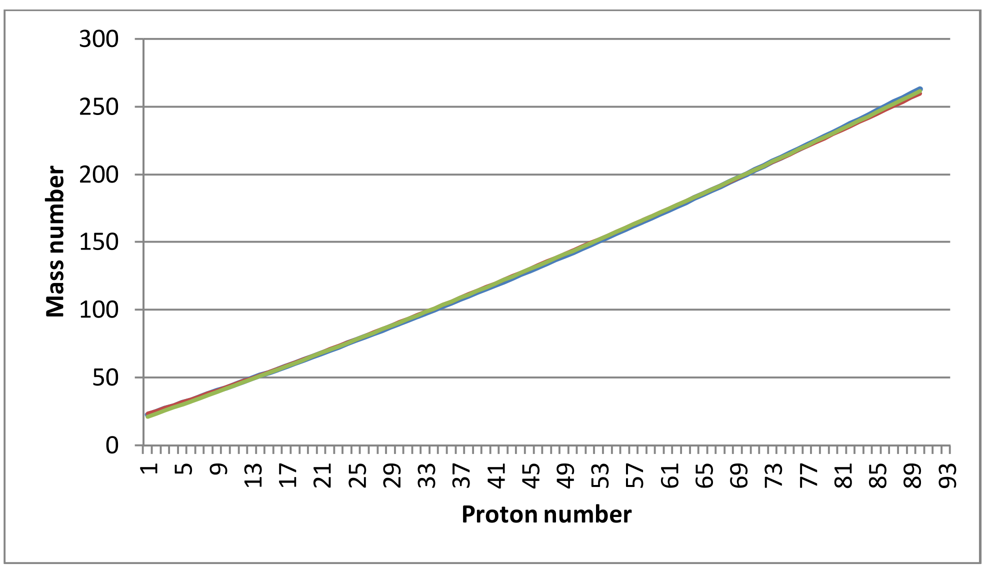
Table-8: To fit the stable mass numbers estimated with relations (6), (19) and (20)

Proton number	Estimated Mass number Relation(6)	Estimated Mass number Relation(19)	Estimated Mass number Relation(20)
11	22.8	23.5	21.2
12	24.9	25.6	23.4
13	27.1	27.7	25.7
14	29.2	29.8	28.0
15	31.4	31.9	30.3
16	33.6	34.0	32.6
17	35.8	36.2	34.9

18	38.1	38.4	37.2
19	40.3	40.6	39.5
20	42.5	42.8	41.9
21	44.8	45.1	44.2
22	47.1	47.4	46.6
23	49.3	49.7	49.0
24	51.6	52.0	51.3
25	54.0	54.3	53.7
26	56.3	56.6	56.1
27	58.6	59.0	58.5
28	61.0	61.4	61.0
29	63.3	63.8	63.4
30	65.7	66.2	65.8
31	68.1	68.6	68.3
32	70.5	71.0	70.7
33	72.9	73.5	73.2
34	75.3	75.9	75.7
35	77.8	78.4	78.2
36	80.2	80.9	80.7
37	82.7	83.4	83.2
38	85.1	85.9	85.7
39	87.6	88.5	88.3
40	90.1	91.0	90.8
41	92.6	93.5	93.3
42	95.2	96.1	95.9
43	97.7	98.7	98.5
44	100.3	101.3	101.1
45	102.8	103.9	103.7
46	105.4	106.5	106.3
47	108.0	109.1	108.9
48	110.6	111.7	111.5
49	113.2	114.4	114.1
50	115.8	117.0	116.8
51	118.5	119.7	119.4
52	121.1	122.3	122.1
53	123.8	125.0	124.8
54	126.5	127.7	127.5
55	129.1	130.4	130.1
56	131.9	133.1	132.8
57	134.6	135.8	135.6
58	137.3	138.5	138.3
59	140.0	141.3	141.0
60	142.8	144.0	143.8
61	145.6	146.8	146.5
62	148.3	149.5	149.3
63	151.1	152.3	152.1
64	153.9	155.1	154.8
65	156.7	157.9	157.6
66	159.6	160.7	160.5
67	162.4	163.5	163.3
68	165.3	166.3	166.1
69	168.1	169.1	168.9
70	171.0	171.9	171.8

71	173.9	174.7	174.6
72	176.8	177.6	177.5
73	179.7	180.4	180.4
74	182.7	183.3	183.3
75	185.6	186.2	186.2
76	188.6	189.0	189.1
77	191.5	191.9	192.0
78	194.5	194.8	194.9
79	197.5	197.7	197.8
80	200.5	200.6	200.8
81	203.5	203.5	203.8
82	206.6	206.4	206.7
83	209.6	209.3	209.7
84	212.7	212.3	212.7
85	215.7	215.2	215.7
86	218.8	218.1	218.7
87	221.9	221.1	221.7
88	225.0	224.0	224.7
89	228.1	227.0	227.8
90	231.3	230.0	230.8
91	234.4	232.9	233.9
92	237.6	235.9	237.0
93	240.7	238.9	240.0
94	243.9	241.9	243.1
95	247.1	244.9	246.2
96	250.3	247.9	249.3
97	253.6	250.9	252.4
98	256.8	253.9	255.6
99	260.0	256.9	258.7
100	263.3	260.0	261.9

Figure 1: Comparison of stable mass numbers estimated with relations (6), (19) and (20)



7. Discussion & Conclusion

1. From the above data, it is possible to conclude that, nuclear binding energy can be addressed with a single energy coefficient [14].
2. Based on relation (14), for $Z \geq 26$, binding energy per nucleon can be expressed as:

$$B_A \cong \left\{1-\left[\frac{kN}{A}\left(\frac{kAA_s}{\sqrt{Z}}+1\right)\right]\right\}\times 8.9 \text{ MeV} \tag{21}$$

3. Based on the new integrated model proposed by N. Ghahramany et al. [10,11,12], it is possible to show that, $Z \cong (40 \text{ to } 83)$, close to the beta stability line,

$$(B)_{A_s} \cong \left[A_s-\left(\frac{N_s^2-Z^2}{3Z}\right)\right]\times 9.5 \text{ MeV} \cong \left[A_s-\left(\frac{kZA_s}{3}\right)\right]\times 9.5 \text{ MeV}$$
$$\text{where, } \left[\frac{N_s^2-Z^2}{Z}\right]\cong kZA_s \tag{22}$$

4. Our new ideas make it possible to study the semi empirical mass formula and strong interaction in a unified way.
5. Considering even-odd corrections and root mean square charge radii of light atomic nuclides, further improvements can be made and better results can be achieved.
6. Arbitrariness in choosing the binding energy coefficients can be eliminated.

Even though some % error is persisting in the proposed binding energy relations, qualitatively they are very simple to follow. Understanding and estimating nuclear binding energy with strong interaction concepts still stands as a really challenging task. So far no such model is available in physics literature. We believe that, results obtained from above relations are simple to understand and seem to be more physical and relatively closer to the experimental data. We are working on deriving the above relations and confident to say that, by following the proposed kind of semi empirical relations, one can certainly implement the strong coupling constant in nuclear binding energy scheme successfully. With further research, a clear, simple and realistic nuclear model pertaining to strong interaction can be developed.

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