Exploration

A Quantum Mechanical View of the Precession of Mercury's Orbit

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Abstract

The calculation of Mercury's precession about the Sun presented in this article offers a quantum mechanical alternative to the calculation based on general relativity (*ref 1*). Further, this alternative provides a plausible explanation of Dark Matter in which gravitons act collectively as a small planet between Mercury and the Sun. This unseen graviton bundle of a planet influences the orbit of Mercury causing its precession.

Keywords: Dark Matter, Dark Energy, universe, expansion, gravity, quantum mechanics.

In terms of the most astonishing fact about which we know nothing, there is Dark Matter and Dark Energy. We don't know what either of them is. Everything we know and love about the universe and all the laws of physics as they apply, apply to four percent of the universe. That's stunning - Neil deGrasse Tyson (ref 2)



The diagram (above left) accentuates the elliptical nature of Mercury's orbit. The diagram above right is a scaled orbit diagram (*ref 3*), that shows how close to a circle the orbit of Mercury actually is.

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Also, it should be noted that the precession phenomena is too small to see over a single orbit. It takes hundreds of orbits to even begin to see this precession. At left is a very, very exaggerated view of Mercury's precession about the Sun.

A. Consideration of A new definition of the graviton

- 1. The graviton is the mediator particle of the gravitational force in the sense that the photon is the mediator particle of the electromagnetic force.
- 2. The photon is the smallest quantum of energy at a given wavelength. The graviton is the smallest quantum of mass at a given distance.
- 3. When gravitons connect masses that are light years apart, they should be as undetectable as photons with wavelengths that are light years long. We may not be able to get photons that are light years long, but this is no limit for gravitons.
- 4. Photons can act as if they had mass when they push solar sails, and when they are reflected between mirrors. (*ref 4*) This has not turned photons into gravitons, however it gives them some of the properties of gravitons.
- 5. It seems not too far a stretch that to think a single graviton is reflected to and fro between masses. This graviton is not something separate from observable mass, but comes packaged with observable mass.

B. Dark Energy and Dark Matter



Dark Energy is hypothesized as the accumulation of graviton mass that is not rotating about an observable mass. It is observable in the universe at the largest scales. See paper: "The Geometry of Dark Energy" (*ref 5*).

Dark Matter is hypothesized as the accumulation of graviton mass that is rotating about observable mass. It is found mostly in rotating galaxies. The precession of Mercury's orbit, considered here, is the simplest example of gravitons acting as Dark Matter.

The three terms, *Dark Energy, Dark Matter*, and *Gravitons*, have overlapping meanings and are somewhat confusing. Things are a little clearer when we speak of graviton mass accumulations in astronomical settings as being in the configurations of either Dark Energy Gravitons or Dark Matter Gravitons. This paper could be called a prequel to the *Prespacetime Journal* paper titled "The Geometry of Dark Energy" (*ref 5*) which concentrates primarily on gravitons acting as Dark Energy.

C. The Insights

This quantum theory of the graviton will be shown to calculate the correct precession of the orbit of Mercury (and Venus and Earth). The math is simple algebra; the physics is introductory, and the astronomy is basic. The insights themselves are not obvious and will raise some questions, which I will anticipate with some Q&A at the end of each insight.

- 1) The first insight was that the graviton is a quantum of mass.
 - a. A single photon has the energy $E = hf = hc/\lambda$, where f is the frequency of the photon, h is Planck constant, c is the speed of light, and λ is the wavelength. Note that in this paper both the Planck constant h will be used as well as the reduced Planck constant h.
 - b. The equation that converts energy to its mass equivalent is $E = mc^2$, where m is the mass of the graviton. This assumes that the momentum of the mass is negligible.
 - c. Equating the two equations in a and b above, we get: $E = hf = hc/\lambda = mc^2$.
 - d. Solving for m, we get: $\mathbf{m} = \mathbf{h}/\lambda \mathbf{c}$. This is the quantum of mass that is the graviton.
 - e. A graviton spans the distance of separation d between two observable mass objects. This distance is the wavelength of the graviton. Thus the graviton is defined by the equation $\mathbf{m} = \mathbf{h}/\mathbf{dc}$. At the astronomical scales of galaxies and the universe as a whole, this mass will take the form of either Dark Matter or Dark Energy. At our solar system scale this dark matter graviton mass causes the precession of Mercury.

Question 1: If the graviton fits the Planck-Einstein equation (E = hf), then it must be a photon?

Answer: The graviton is like a photon, but a photon that cannot move freely because it is tethered at both ends. And yes, this is just a postulate that we can test to see if it makes sense.

Question 2: What is the graviton tethered between?

Answer: For Dark Energy gravitons (where masses move in straight lines with respect to each other) the tether points are Planck masses defined using the Planck constant h (Pm = $(hc/G)^{0.5}$. This result was worked out in a previous paper in *Prespacetime Journal*, (*ref 2*).

For Dark Matter gravitons (where masses revolve about each other) the tether points are Planck masses defined using the reduced Planck constant h. This is the agreed upon definition of a Planck mass (Pm = (hc/G)^{0.5} (*ref 6*). This tethering will be considered below in the second insight.

Question 3: Why are there two types of gravitons? **Answer:** It seems that rotary motion forces the gravitons to spread out. See diagram below.

Note that: $h=2\pi h$, where h is the Planck constant and h is the reduced Planck constant.

2) The second insight: Planets that orbit the Sun are connected to the Sun by a lot of gravitons the number of which (N) can be calculated.



a. The kinetic energy of a planet that is rotating about the Sun.

This energy is usually thought of as $\frac{1}{2}$ mv². However, it can also be thought of as a force times a distance as diagramed left. Note that the wavelength of the graviton in this situation is $2\pi d$, the circumference of the orbit. The force is toward the Sun and the motion is along the circumference.

b. The energy in the orbit of the planet (mass m) is $2\pi d(GMm/d^2)$. This energy in the orbiting planet can be equated to the energy of each graviton (hc/d as explained above) times the number of gravitons N.

force	Х	distance	= N (hc/d)
$[GMm/d^2]$	Х	[2πd]	= N (hc/d)

[GMm/d] $[2\pi] = N$ (hc/d) Solving for N we get: N = $[2\pi GMm/d]$ [d/hc] Simplifying we get: N = $2\pi GMm/hc$ or N = GMm $2\pi/hc$ or N = Mm/(hc/ $2\pi G$) Since h/ $2\pi = h$, we get N = Mm/(hc/G) and since Pm² = hc/G, we get N = Mm/Pm²

The result $N = Mm/Pm^2$ can be written as N = [M/Pm][m/Pm]

In words: the number of gravitons connecting the masses M and m is the mass M divided by the Planck mass times the mass m divided by the Planck mass.

This is a very interesting result. It indicates that the Planck mass is the minimum mass that can support a Dark Matter graviton. It also indicates that dark matter gravitons are tethered between Planck masses.



Question 3: Does this mean that there is no gravitational mass below the Planck mass? **Answer:** This exploration points to that conclusion. However, this does not eliminate inertial mass from existing below the Planck mass.

Question 4: Can we create gravitons in the lab?

Answer: Until we can create matter in the lab that has a mass greater than the Planck mass, we will not be able to create gravitons. Note that physicists have produced electron-positron pairs from gamma rays, but this mass is well below the needed Planck mass.

Question 5: Is there a way of detecting gravitons in the lab?

Answer: I expect that if we had several physical weights and put them together we would measure a gravitational mass that is greater than the sum of the individual weights when they are separated. I think we can test this idea with something like a modified torsional balance of the type Cavendish used to measure the gravitational constant G (*ref 7*).

D. Graviton Summary

- a. The number of gravitons N connecting two masses is: $N = M_1 M_2 / P_M^2$
- b. Each Planck mass in M₁ connects to every Planck mass in M₂.
- c. Each Planck mass in M₂ connects to every Planck mass in M₁.
- d. Each graviton produces a gravitational force of $hc/(2\pi d^2)$
- e. Each graviton connecting M1 to M2 has a mass of m=h/dc
- f. The total graviton mass connecting M₁ to M₂ is: N (h/dc) = $(M_1M_2/P_M^2)(h/dc)$

Why should the Planck Mass be such a fundamental quantity? The Planck mass approximates the mass of a small eyelash. This is much larger than any atom or molecule. Why should this ordinary observable mass be special?

Perhaps the answer lies in history. If we assume that matter arrived shortly after the Big Bang, this matter would have been very compact. Something that now has the mass of a Planck mass may have been compacted to the extent that it was a mini black hole. When the universe expanded, the mini Planck masses expanded into ordinary matter. This would be similar in nature to the initial radiation in the universe becoming the cosmic background radiation.

What happens to masses less than the Planck Mass? This theory has no gravitons connecting to particles, atoms, and molecules that are less than the Planck Mass. What does this mean? Richard Muller provides an interesting insight by way of the Higgs field (*ref 8*):

In the theory of the Big Bang, it proved difficult to include particles that had mass. So some theorists looked to see what would happen if all particles had zero mass. The theory worked! Wow! But it didn't represent reality.

Then Higgs and his colleagues came up with a fascinating way to handle it. What if electrons, quarks, all the elementary particles were indeed massless? Maybe they are! But that contradicts reality. We can measure the mass of the electron.

But they knew that the apparent mass of the electron changes when it moves through a crystal (like silicon or quartz). So maybe the mass of the electron is an illusion. Electrons behave as if they have mass, but they don't really. It's just that they are moving through a crystal-like substance.

This crystal-like substance eventually was called the "Higgs field." It fills all of space, and that's why particles behave as if they had mass.

What about the early universe? Theorists had already speculated about the creation of new fields, and the Higgs field fit right in. In the early universe, the Higgs field was zero, everywhere. Then there was a "phase transition" in which through "spontaneous symmetry breaking" the Higgs field would rapidly grow and reach a constant value, the value it has now. So from that time onward, all particles would have an apparent mass.

It is just speculation, but it is possible that gradients of gravitons can act like a prism (*ref 5*). Could it be that the graviton filled space of the universe is the Higgs field?

E. A third insight

This third insight was that the Dark Matter gravitons between Mercury and the Sun could produce an angular momentum that would add to Mercury's angular momentum, causing a slight overshoot in the orbit beyond the preceding 360-degree orbit. The ratio describing this angular momentum change over an orbit would be:

 $\frac{\Delta \text{Deg}}{360} = \frac{\text{Angular Momentum of Dark Matter Mass}}{\text{Angular Momentum of Mercury Mass}}$

Where ΔDeg is the precession in degrees (eventually to be converted to arc-seconds). Angular Momentum is defined as the mass of an object times its tangential velocity; and Angular Velocity is defined as the rotational speed. The graviton bundles that connect to a planet have the same rotational speed as the planet itself.

This equation is a little tricky because the Dark Matter gravitons have changing mass over the orbit (m = h/dc) because the orbit is not a perfect circle. We can handle this with two methods. The first method is to average the aphelion and the perihelion distance of Mercury (called the semi-major axis) and use it as a constant radius over an orbit. The second method is to divide the 360-degree orbit into smaller segments (each with its own distance) and calculate the precession of each segment. The total precession would be the sum of all the precession segments.

The precession (ΔDeg) was calculated using both methods. The first method, calculating the distance d as the average of the perihelion and aphelion gave a precession of 43.2 arc-seconds. The second method using 10-degree increments gave a precession of 45 arc-seconds (using a ruler and protractor to measure the distance of Mercury at each orbit position given in the diagrams of Mercury's orbit (*ref 3*).

The calculation that follows uses the first method (figure bottom left). This method resulted in 43.2 arc-seconds, which agrees very well with measured precession values. It is also much easier to calculate.



Why should the average of the perihelion and aphelion distances work to compensate for mass changes over an orbit?

Answer: The length of a semi-major axis is often termed the size of the ellipse. It can be shown that the average separation of a planet from the Sun as it goes around its elliptical orbit is equal to the length of the semi-major axis. Thus, by the "radius" of a planet's orbit one usually means the length of the semi-major axis (ref 9).

Following this lead, the Dark Matter line of gravitons moving with Mercury can be considered to be a planet. The orbit of this planet is a circle that has a constant radius "d" that is the average of the perihelion and aphelion distances (the Semi-Major Axis) of Mercury.



F. Setting up a ratio to Solve for the precession:

1. $\frac{\Delta \text{Deg}}{360} = \frac{\text{Angular Momentum of } M_{\text{DM}}}{\text{Angular Momentum of Mercury Mass}}$ M_{DM} is the Dark Matter mass. M_{Mercury} is the mass of Mercury.

The graviton bundle moves like a rigid rod that rotates with Mercury about the Sun. Thus the angular velocity of the Dark Matter and Mercury are the same.

The velocity of a rotating object is defined as the angular velocity times the radius. The angular momentum is the mass of the object times the velocity of the object.

2. $\Delta \text{Deg} = (\text{Speed of } M_{\text{DM}}) (M_{\text{DM}}) = (d/2)(\text{Angular Velocity of } M_{\text{DM}})(M_{\text{DM}})$ (d)(Angular Velocity of $M_{\text{Mercury}})(M_{\text{Mercury}})$

Where d is the distance to Mercury and d/2 is the distance to the center of M_{DM}.

3. Since the angular velocity of both M_{DM} and M_{Mercury} are equal (see diagram above) we get:

 $\frac{\Delta \text{Deg}}{360} = \frac{(0.5) (\text{M}_{\text{DM}})}{\text{M}_{\text{Mercury}}}$

- 4. We now have the precession in a simple form: $\Delta Deg = 180 [M_{DM}/M_{Mercury}]$
- 5. However, we still need to get more specific about M_{DM} . M_{DM} is the mass of each graviton times the number of gravitons as shown earlier in section (D-f).



 $M_{DM} = (M_{Mercury}M_{Sun}/P_{M}^{2})(h/dc)$ With some cleanup: $M_{DM} = \frac{hM_{Sun}M_{Mercury}}{dcP_{M}^{2}}$

This d is now the semi-major axis shown as davg in the diagram.

6. We can put the value of M_{DM} into the equation $\Delta Deg = 180 [M_{DM}/M_{Mercury}]$ and get:

$$\Delta Deg = \frac{(180) h M_{Sun} M_{Mercury}}{(dc P_M^2) (M_{Mercury})} = \frac{(180) h M_{Sun}}{dc P_M^2}$$

7. We can now substitute ($\underline{h}c/G$) for P_M^2 and get

$$\Delta \text{Deg} = \underbrace{(180) \text{ hG } \text{M}_{\text{Sun}}}_{\text{dhc}^2} \qquad \text{Since h/h} = 2\pi \qquad \Delta \text{Deg} = \underbrace{2\pi(180) \text{ G } \text{M}_{\text{Sun}}}_{\text{dc}^2}$$

$$\Delta Deg = \underline{1130.97 \text{ G } M_{Sun}}{dc^2}$$
 The precession equation for an orbit of Mercury

This equation could have been developed using either the Planck mass (P_M) or G. The gravitational constant G was chosen because it is a more familiar quantity.

Note that this resulting equation gives no indication that Dark Matter gravitons were a necessary part of its development. The equation seems to echo the general relativity result that it is only the Sun's mass and distance from the planet that determines the amount of precession. This equation hides our knowledge that Dark Matter was involved!

More about how this theory compares with general relativity will be considered shortly.

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G. Solving the equation: $\Delta Deg = \frac{1130.97 \text{ G M}_{\text{Sun}}}{\text{d}c^2}$

Note that this equation for the precession ΔDeg is not specific for Mercury even though we started out analyzing Mercury's orbit.

Constants:	Gravitational Constant G:	$6.674 imes 10^{-11}$
	<u>"M_{Sun}" Mass of Sun:</u>	$1.989 \times 10^{30} \text{ kg}$
	"c" Speed of light:	2.99×10^8 m/s sep
	Speed of light squared:	$8.94 imes 10^{16}$

Variables: d: The Semi-Major Axis in meters, a variable depending upon the planet. R_{Davs}: Days in an Earth year divided by the days in a Planet's orbit. We need this factor since we measure precession with respect to the Earth.

1. $\Delta Deg = (1130.97)(G)(M_{Sun}) / (dc^2)$ the starting equation for precession = (1130.97)(6.674 x 10⁻¹¹)(1.989 × 10³⁰) / [(d)(8.94 × 10¹⁶)][SEP]SEP] = $(1/d)(1.679 \times 10^6)$ degrees for each orbit of the planet.

We can convert degrees to arc-seconds by multiplying by 3600 to get $\frac{1}{\text{sep}}$:

 $\Delta \text{Deg} = (1/d)(1.6749 \text{ x } 10^6)(3600) = (1/d)(6.02964 \text{ x } 10^9)$ arc-seconds per orbit of planet. 2.

We need to multiply the above term by R_{Days} to get precession per orbit of Earth.

 $\Delta Deg = (R_{Days}) (1/d)(6.02964 \times 10^9)$ arc-second of precession for each orbit of Earth. 3.

To get the result in arc-seconds per Earth century we need to multiply by 100.

4. $\Delta Deg = (R_{Davs}/d)(6.02964 \times 10^{11})$ arc-second for each Earth century.

We can now create a table comparing the precession of Mercury, Venus, and Earth.

H. Table of Precessions created using $\Delta Deg = (R_{Davs}/d)(6.02964 \times 10^{11})$

Comparison	of Precessi	ons of Mercury,	Venus, and Earth
(Note 1: all pre	cessions are i	n arc-seconds per	century)
(Note 2: for pla	nets beyond E	Earth, precession is	to small to observe)
	Observed	General Relativity	QM Graviton Theory
	Precession	Calculated	Calculated
Mercury	43.1 ± 0.5	43	43.2
Venus	8.4 ± 4.8	8.6	9.0
Earth	5 ± 1.2	3.8	4.0

Precession Related Information About Mercury, Venus, and Earth (QM Graviton Theory)							
Planet Mass	Dark Matter (kg)	Orbit in	Ratio: 365.25 days	Semi-Major	Precession		
(kg)	Graviton Bundle to Sun	Earth Days	to OED days	Axis (m)	(arc-seconds per century)		
		(OED)	(R _{Days})	(average distance to Sun)			
3.30 x 10 ²³	5.31 x 10 ¹⁶	87.97	4.15	5.79 x 10 ¹⁰	43.2		
4.87 x 10 ²⁴	4.19 x 10 ¹⁷	224.7	1.63	10.821 x 10 ¹⁰	9.0		
5.97 x 10 ²⁴	3.71 x 10 ¹⁷	365.256	1.00	14.96 x 10 ¹⁰	4.0		
	Planet Mass (kg) 3.30 x 10 ²³ 4.87 x 10 ²⁴	Planet Mass (kg) Dark Matter (kg) 3.30 x 10 ²³ 5.31 x 10 ¹⁶ 4.87 x 10 ²⁴ 4.19 x 10 ¹⁷	Planet Mass (kg) Dark Matter (kg) Orbit in 3.30 x 10 ²³ Graviton Bundle to Sun (OED) Earth Days (OED) 4.87 x 10 ²⁴ 4.19 x 10 ¹⁷ 224.7	Planet Mass (kg) Dark Matter (kg) Orbit in Earth Days (OED) Ratio: 365.25 days to OED days (OED) 3.30 x 10 ²³ 5.31 x 10 ¹⁶ 87.97 4.15 4.87 x 10 ²⁴ 4.19 x 10 ¹⁷ 224.7 1.63	Planet Mass (kg) Dark Matter (kg) Orbit in Earth Days Ratio: 365.25 days to OED days Semi-Major 3.30 x 10 ²³ 5.31 x 10 ¹⁶ 87.97 4.15 5.79 x 10 ¹⁰ 4.87 x 10 ²⁴ 4.19 x 10 ¹⁷ 224.7 1.63 10.821 x 10 ¹⁰		

I. Table of values for Mercury, Venus and Earth

J. Graviton Theory vs. General Relativity Theory

Both theories can calculate the precession of Mercury, Venus and the Earth to good accuracy. The likelihood of this is remote without a strong connection between the two theories. I do not know what the connection is; however, I can point out the differences between the two theories.

a. This graviton theory has Newton's law of gravity as fundamental even though it needs to be compensated for a Dark Matter graviton effect, which imitates a planet orbiting within the orbit of Mercury.

In contrast, general relativity has the curvature of space-time as fundamental and posits that Newton's law of gravity is an approximation when speeds are slow and masses are small.

b. This graviton theory was developed from quantum mechanical concepts and has the graviton as a quantum of mass that corresponds to the photon's quantum of energy.

General relativity does not at present have a connection to quantum mechanics.

c. The concept of Dark Matter gravitons developed here; in conjunction with the Dark Energy gravitons developed previously (*ref 5*), accounts for Dark Energy and Dark Matter.

General relativity at present does not account for either Dark Energy or Dark Matter.

d. This graviton theory cannot as yet account for the curvature of space-time effect measured by Sir Arthur Eddington showing that the sun can curve the light from other stars (*ref 10*).

General relativity predicts and calculates the bending of starlight by observable masses.

e. General relativity says that acceleration of a mass in free space is equivalent to gravity.

This graviton theory has no free space; an all-encompassing massy network connects the

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stuff of the universe. For an object to move (accelerate) it has to lengthen and shorten its attached graviton bundles. This makes the background of space a fundamental component of motion.

f. Gravitons are matter waves. These gravitons connect all the masses in the universe. Colliding masses could produce changing patterns on an existing sea of graviton waves. We can think of this as gravity waves. This is a speculation without experimental verification.

General relativity has space-time that connects all the masses in the universe. Colliding masses are thought to produce propagating waves on this existing pattern of curved space-time. Experiments have been made that support this theory.

K. Parting Thoughts

In spite of what looks like insurmountable obstacles, this graviton theory and general relativity may be identical if it can be shown that:

- **a.** Instead of mass curving space-time directly, mass does it indirectly through gravitons. Gradients of gravitons may bend light; similar to the way a prism bends light. (*ref 5*).
- **b.**A mechanism can be found in general relativity that would explain Dark Energy and Dark Matter. Some very competent people are working on this.

French astronomer Urbain Le Verrier predicted that a planet (Vulcan) was causing Mercury's precession (*ref 11*). According to Tom Levenson, "Vulcan is remarkable because the idea of this little body inside the orbit of Mercury makes perfect sense," (*ref 12*). The work presented here shows that Le Verrier was on the correct track. There is a mass between Mercury and the Sun. This mass consists of a bundle of long-wavelength gravitons with a mass value of 05.3×10^{16} kg.

Dark Matter is very strange. It is a line of mass (a bundle of gravitons) that connects Mercury and the Sun and has a mass of about a million kg per meter for 5.76×10^{10} meters. And the question is, why can't we see this huge mass directly and can only detect it via the subtle phenomena of precession? I believe the answer is that this graviton mass is a wave phenomenon and manifests quite differently than observable mass such as a golf ball or planet; and to detect it we need to couple to it. This is similar to radio reception. Even if we had a radio transmitter that could produce 1 megawatt of power at a wavelength of 1000 kilometers, we cannot sense this transmission with an antenna that is only 1 meter long; it is not capable of coupling to the energy. However, if we had an antenna that was 1000 kilometers long, we could easily couple to this energy.

The graviton waves we are considering for Dark Matter and Dark Energy behave in a fashion similar to electro-magnetic waves (photons), even though they are not exactly photon electro-magnetic phenomena. We cannot directly sense the graviton mass connecting Mercury and the Sun because it has a wavelength of 5.76×10^{10} meters and a period of 3.23 minutes. This

low frequency is below the ELF band (Extremely Low Frequency) and would be in the humorously postulated *Ludicrously Low Frequency* (LLF) band. We haven't given much thought on how to couple to this type of electromagnetic like energy. However, the Mercury-Sun combination has the correct distance for coupling to this immense graviton mass.

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