Article

On the Saha's Generating Function for the Hermite Polynomials

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Abstract

We give an elementary deduction of the Saha's expression to generate the Hermite polynomials.

Keywords: Hermite polynomials, Saha's generating function.

1. Introduction

The Hermite polynomials $H_n(x)$ [1-3] can be generated via the expression:

$$\exp(2y\,z - z^2) = \sum_{n=0}^{\infty} \frac{z^n}{n!} H_n(y), \tag{1}$$

but Saha [4] obtained the following alternative relation to construct these polynomials:

$$\exp(2x\,\eta - \eta^2) = \sum_{n=0}^{\infty} \frac{[\left(\gamma - \sqrt{\gamma^2-1}\right)\eta]^n}{n!} H_n\left[\left(\sqrt{\gamma^2-1}+\gamma\right)x - \eta\sqrt{\gamma^2-1}\right], \quad |\gamma| \leq 1. \tag{2}$$

Here we employ (1) to give an elementary deduction of this Saha's result.

2. Saha's generating function

We introduce the variables x and η such that:

$$y = i\left(x\,e^{-i\varphi} - \eta\cos\varphi\right), \qquad z = -i\,\eta\,e^{i\varphi}, \tag{3}$$

where φ is arbitrary. Then it is easy to see that $2y\,z - z^2 = 2x\,\eta - \eta^2$, hence (1) takes the form (2) if we use the notation $\gamma = \sin\varphi$; thus we observe that (2) contains to (1) for $\varphi = \pi/2$.

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In [5] exists other deduction of (2) via an expression between Laguerre and Hermite polynomials obtained by Talman [6] employing Group theory. Let's remember that the study of formulae involving Hermite polynomials has great importance in the analysis of several quantum mechanical problems [3, 7, 8].

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